Change:

$H_1 = 4.2$ ft

$H_2 = 4.9$ ft

$H_2 - H_1$

Feb 11-6:04 PM

$L(t) = t^2 - 5t$

t	0	1	2	3	4	5
$L(t)$	0	-4	-6	-6	-4	0

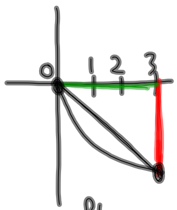
v_{av} between $t=0$ and $t=3$

Feb 11-6:08 PM

$v_{av} = \frac{\text{distance traveled}}{\text{time}}$

$= \frac{L(3) - L(0)}{3-0}$

$= \frac{-6-0}{3} = \frac{-6}{3} = -2\ \frac{ft}{s}$



Feb 11-6:09 PM

Uniform motion has a linear location function

$L(t) = 3t - 5$

v_{av} between $t=2$ and $t=10$

$v_{av} = \frac{L(10) - L(2)}{10-2} = \frac{25-1}{8} = 3$

Feb 11-6:13 PM

$t=3$ and $t=8$ $\quad L(t) = 3t-5$

$v_{av} = \frac{L(8) - L(3)}{8-3} = \frac{19-4}{5} = 3$

Feb 11-6:18 PM

$L(t) = -t^2 + 12t$ in meters, t in seconds

$v_{av} = ?$ between $t = 2s$ and $t = 5s$

$v_{av} = \frac{L(5) - L(2)}{5s - 2s} = \frac{35m - 20m}{3s}$

$= \frac{15m}{3s} = 5\ \frac{m}{s}$

Feb 11-6:20 PM

v_{av} between $t=2s$ and $t=2.5s$

$$v_{av} = \frac{L(2.5)-L(2)}{2.5-2} = \frac{23.75m-20m}{0.5s}$$

$$= \frac{3.75m}{0.5s} = 7.5\frac{m}{s}$$

Feb 11-6:27 PM

v_{av} between $t=2s$ and $t=2.1s$

$$v = \frac{L(2.1)-L(2)}{2.1s-2s} = \frac{20.79m-20m}{0.1s}$$

$$= \frac{0.79m}{0.1s} = 7.9\frac{m}{s}$$

between $t=2$ and $t=2.001$

$$\frac{20.007999-20}{2.001-2} = \frac{0.007999m}{0.001s}$$

$$= 7.999\frac{m}{s}$$

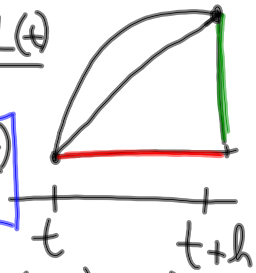
Feb 11-6:29 PM

def: The instantaneous velocity is the limit of the average velocity as the time interval approaches a point.

Feb 11-6:33 PM

$$v_{av} = \frac{L(t+h)-L(t)}{t+h-t}$$

$$\boxed{v_{av} = \frac{L(t+h)-L(t)}{h}}$$



$$v_{ins}^{at\ t} = \lim_{h\to 0}\frac{L(t+h)-L(t)}{h}$$

Feb 11-6:35 PM

$L(t) = -t^2+12t$ L in meters, t in seconds

v_{inst} at $t=2$

$$= \lim_{h\to 0}\frac{L(2+h)-L(2)}{h}$$

$$= \lim_{h\to 0}\frac{[-(2+h)^2+12(2+h)]-(-2^2+12\cdot 2)}{h}$$

$$= \lim_{h\to 0}\frac{[-(h^2+4h+4)+24+12h]-20}{h}$$

$$= \lim_{h\to 0}\frac{-h^2-4h-\cancel{4}+\cancel{24}+12h-\cancel{20}}{h}$$

$$= \lim_{h\to 0}\frac{-h^2+8h}{h} = \lim_{h\to 0}\frac{\cancel{h}(-h+8)}{\cancel{h}}$$

$$= \lim_{h\to 0}(-h+8) = \boxed{8}$$

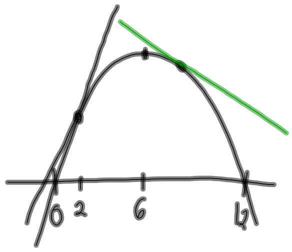
Feb 11-6:39 PM

Geometric meaning:
Slope of the tangent line drawn to $L(t)$ at $t=2$

Feb 11-6:52 PM

v_{inst} at $t=8$

We expect: a negative number but smaller than 8.



Feb 11-6:54 PM

$$\lim_{h\to 0}\frac{L(8+h)-L(8)}{h}$$

$$\lim_{h\to 0}\frac{\left[-(8+h)^2+12(8+h)\right]-\left[-8^2+12\cdot 8\right]}{h}$$

$$\lim_{h\to 0}\frac{\left[-(h^2+16t+64)+96+12h\right]-32}{h}$$

$$\lim_{h\to 0}\frac{-h^2-16h-\cancel{64}+\cancel{96}+12h-\cancel{32}}{h}$$

$$\lim_{h\to 0}\frac{-h^2-4h}{h}=\lim_{h\to 0}\frac{\cancel{h}(-h-4)}{\cancel{h}}$$

$$\lim_{h\to 0}(-h-4)=-4$$

Feb 11-6:56 PM

at t

$$\lim_{h\to 0}\frac{L(t+h)-L(t)}{h}$$

$$\lim_{h\to 0}\frac{\left[-(t+h)^2+12(t+h)\right]-(-t^2+12t)}{h}$$

$$\lim_{h\to 0}\frac{-(t^2+h^2+2th)+12t+12h+t^2-12t}{h}$$

$$\lim_{h\to 0}\frac{-\cancel{t^2}-h^2-2th+\cancel{12t}+12h+\cancel{t^2}-\cancel{12t}}{h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(-h-2t+12)}{\cancel{h}}=\lim_{h\to 0}(-h-2t+12)$$

$$=-2t+12$$

Feb 11-7:07 PM

location function $L(t)$	a function $f(x)$
average velocity $\frac{L(t+h)-L(t)}{h}$ slope of secant line	difference quotient $\frac{f(x+h)-f(x)}{h}$ slope of secant line
$\lim_{h\to 0}\frac{L(t+h)-L(t)}{h}$ instantaneous velocity (if exists)	$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ derivative of f at x notation: $f'(x)$ (if exists)

Slope of tangent line drawn to L at t or f at x

Feb 11-7:30 PM

$v(t)$ if exists is the velocity function	$f'(x)$ or the (first) derivative of f

Feb 11-7:37 PM

DON'T EVER FORGET !

1. f' measures the slope of the tangent line
2. f and f' can always be thought of as location and velocity.

Feb 11-7:39 PM

Claim: If $f(x)=c$ for all x, then $f'(x)=0$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{c-c}{h}=\lim_{h\to 0}\frac{0}{h}\quad (h\neq 0)$$
$$=\lim_{h\to 0}0=0$$

Feb 11-7:45 PM

Differentiate by computing the limit of the differential quotient:

1. $f(x)=x^2$
2. $f(x)=\frac{1}{x}$
3. $f(x)=\sqrt{x}$
4. $f(x)=x^3$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$

Feb 11-7:49 PM

① $f(x)=x^2$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\lim_{h\to 0}(2x+h)$$
$$=2x$$

Feb 11-8:10 PM

Feb 11-7:37 PM