Quiz 11 will cover the following material: All material covered in Classes 1-12

The following Sample Quiz is intended to demonstrate the difficulty level of the questions. It is not intended as a comprehensive review or list of the type of questions that can appear on the quiz.

Sample Quiz 11

1. Differentiate $f(x) = \dfrac{1}{2-x}$ by computing the limit of the difference quotient.

2. Compute each of the following limits.

   a) $\lim\limits_{x\to 10^+} \dfrac{x+3}{\sqrt{x-10}}$ c) $\lim\limits_{x\to\infty} \tan^{-1} x$ e) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x}$

   b) $\lim\limits_{x\to\infty} \dfrac{\log_2(3x)}{\log_4(6x)}$ d) $\lim\limits_{x\to\infty} \left(1-\dfrac{1}{x}\right)^{x+3}$ f) $\lim\limits_{x\to 0} \dfrac{\tan^2 3x}{x\sin x}$

3. a) Prove the first version of the Intermediate Value Theorem.

   b) Prove the second version of the Intermediate Value Theorem.

   c) Prove the product rule of derivatives.

   d) Prove that $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

   e) Prove that if f is differentiable at c, then it is continuous there.

   f) Prove Rolle's theorem.

   g) Prove that if $f' = g'$, then $f(x) = g(x) + c$ for some real number c.

4. Differentiate each of the following.

   a) $f(x) = \sin x\left(2x^3 - x^2\right)$ d) $f(x) = \dfrac{x^6 - x^2}{x^4}$ g) $g(x) = \dfrac{x^3 \ln x}{3} - \dfrac{x^3}{9}$

   b) $g(x) = \dfrac{\cos x}{x}$ e) $h(x) = \ln\left(2x^3\right)$ h) $f(x) = \dfrac{\ln x}{x}$

   c) $f(x) = x\ln x - x$ f) $f(x) = x^{10}\sin x$

5. Compute each of the given antiderivatives.

   a) $\displaystyle\int \left(6x - 12x^2\right) dx$ d) $\displaystyle\int x\, dx$ g) $\displaystyle\int \left(x^2 + x + \frac{1}{x} + \frac{1}{x^2}\right) dx$

   b) $\displaystyle\int (\sin x - x)\, dx$ e) $\displaystyle\int y\, dx$

   c) $\displaystyle\int 1\, dx$ f) $\displaystyle\int (\sin x + \cos x)\ dx$

6. Find $s(t)$ and $v(t)$ given that $a(t) = 12$, $v(0) = 9$, and $s(0) = 10$.

7. Find all values of a and b for which the line $y = -3x - 18$ is a tangent line to $f(x) = 5x^3 + ax^2 + bx - 20$ at $x = 1$.

8. Find the number that satisfies the conclusion of the Mean Value Theorem for

   a) $f(x) = x^3 - x$ on $[0, 3]$ b) $f(x) = \log_2 x$ on $[1, 16]$.

9. Suppose that an object's location function is given by $L(t) = t^3 - 9t^2 + 15t + 4$ on domain $[0, 5]$. Find the moment when the object is moving with the greatest speed. What is that greatest speed?

10. Find a degree 4 polynomial $P(x)$ with the following properties: $P(0) = -2$, $P'(0) = 5$, $P''(0) = -10$, $P^{(3)}(0) = 18$ and $P^{(4)}(0) = -48$

11. Find all relative maximums and minimums for $f(x) = 5x^4 - 10x^3 + 2x^5 - 2$.

12. The sum of two positive numbers is 10. What is the greatest possibe value of the product of the square of one number and the cube of the other number?

13. Let $A(6,1)$ and $B(10,3)$. Suppose that C is a point on the parabola $y = x^2$. Find the point C for which the area of triangle ABC is the smallest.

Answers

1. Claim: $\dfrac{d}{dx}\left(\dfrac{1}{2-x}\right) = \dfrac{1}{(x-2)^2}$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{2-(x+h)} - \frac{1}{2-x}}{h} = \lim_{h\to 0} \frac{\frac{1}{2-x-h} - \frac{1}{2-x}}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\left(\frac{2-x}{(2-x-h)(2-x)} - \frac{2-x-h}{(2-x-h)(2-x)}\right) = \lim_{h\to 0} \frac{1}{h}\cdot\frac{2-x-(2-x-h)}{(2-x-h)(2-x)} \\ &= \lim_{h\to 0} \frac{1}{h}\cdot\frac{2-x-2-x+h}{(2-x-h)(2-x)} = \lim_{h\to 0} \frac{h}{h(2-x-h)(2-x)} \\ &= \lim_{h\to 0} \frac{1}{(2-x-h)(2-x)} = \frac{1}{(2-x)^2} = \boxed{\frac{1}{(x-2)^2}} \end{aligned}$$

2. a) ∞ b) 2 c) $\dfrac{\pi}{2}$ d) $\dfrac{1}{e}$ e) 0 f) 9 3. see handouts

4. a) $\cos x\left(2x^3 - x^2\right) + \sin x\left(6x^2 - 2x\right)$ b) $-\dfrac{1}{x}\sin x - \dfrac{1}{x^2}\cos x$ c) $\ln x$ d) $2x + \dfrac{2}{x^3}$ e) $\dfrac{3}{x}$
f) $10x^9 \sin x + x^{10}\cos x$ g) $x^2 \ln x$ h) $\dfrac{1}{x^2} - \dfrac{1}{x^2}\ln x$

5. a) $-4x^3 + 3x^2 + C$ b) $-\cos x - \dfrac{1}{2}x^2 + C$ c) $x + C$ d) $\dfrac{1}{2}x^2 + C$ e) $xy + C$ f) $\sin x - \cos x + C$
g) $\dfrac{1}{3}x^3 + \dfrac{1}{2}x^2 + \ln|x| - \dfrac{1}{x} + C$

6. $v(t) = 12t + 9$ $s(t) = 6t^2 + 9t + 10$ 7. $a = -12, b = 6$ 8. a) $\sqrt{3}$ b) $\dfrac{15}{4\ln 2}$ 9. $f'(0) = 15$

10. $P(x) = -2x^4 + 3x^3 - 5x^2 + 5x - 2$ 11. relative minimum: $(1,-5)$ relative maximum: $(-3,187)$

12. 3456 13. $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$

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