Students must be able to correctly **state** the following theorems
Completeness Axiom, Intermediate Value Theorem (both forms), Extreme Value Theorem, Rolle's Theorem, Mean Value Theorem

Students must be able to prove the following theorems:

Differentiating functions using the definition (limit of the differential quotient); $\lim_{x\to 0^+}\frac{\sin x}{x}=1$; $\frac{d}{dx}(\sin x)=\cos x$ and $\frac{d}{dx}(\cos x)=-\sin x$ and $\frac{d}{dx}(\ln x)=\frac{1}{x}$; If a function is differentiable at a number x, then it is continuous there. The product rule and quotient rule for derivatives; The Intermediate Value Theorem, The Mean Value Theorem.

The following Sample Quiz is intended to demonstrate the difficulty level of the questions. It is not intended as a comprehensive review or list of the type of questions that can appear on the quiz.

Sample Quiz 13

1. Compute each of the following limits.

   a) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$ b) $\lim_{x\to 3^+}\frac{|x-3|}{x-3}$ c) $\lim_{x\to 5}\frac{\sqrt{x-5}}{x-5}$ d) $\lim_{x\to\pi}\frac{1+\cos x}{(x-\pi)^2}$ e) $\lim_{x\to 0}\frac{\sin 5x}{\tan 3x}$

2. Differentiate each of the following.

   a) $f(x)=\sin x-x\cos x$ c) $f(r)=\tan r$ e) $f(x)=\sqrt{x^4+3x^2+2}$ g) $f(x)=\ln(\sec x)$

   b) $f(x)=\cos^2 x$ d) $f(x)=\frac{\ln x}{x^2+1}$ f) $f(x)=\frac{1}{(3x-5)^8}$

3. Compute each of the given antiderivatives.

   a) $\int\left(x-\frac{1}{x}\right)dx$ b) $\int\left(x^2+\sqrt{x}+\frac{1}{\sqrt{x}}+\frac{1}{x^2}\right)dx$ c) $\int\sin x dx$

4. Find all relative and absolute maximums and minimums for each of the following functions.

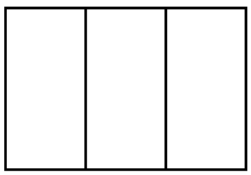
   a) $f(x)=x^4-8x^2+5$ on $[-3,2]$ b) $f(x)=x^5-5x^4+5x^3+2$ on $[-1,3]$

5. Find all values of c that satisfy the conclusion of the Mean Value Theorem.

   a) $f(x)=x^3-x$ on $[-2,6]$ b) $f(x)=\sqrt{x}$ on $[0,4]$ c) $f(x)=\sin x$ on $[0,\pi]$ d) $f(x)=\frac{1}{x}$ on $[1,5]$

6. We know the following about an object: it has a constant acceleration of $-2\frac{\text{m}}{\text{s}^2}$, at $t=5\,\text{s}$ it had a velocity of $4\frac{\text{m}}{\text{s}}$, and at $t=10\,\text{s}$ it had a location of $120\,\text{m}$. What is the initial velocity and initial location of the object?

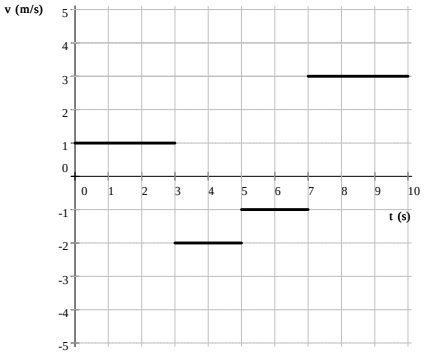
7. We want to create three adjacent rectangular enclosings as shown on the figure. The total area is to be $60\,\text{m}^2$. What dimensions will require the least amount of fencing?



8. A rectangular box, open at the top, is to be constructed from a rectangular sheet of cardboard $18\,\text{cm}$ by $36\,\text{cm}$ by cutting out equal squares in the corners and folding up the sides. What sides squares should be cut out for the container to have maximal volume?

9. Consider all lines with negative slopes that pass through the point $P\,(6,2)$. Let us denote the origin by O, the $x-$intercept of the line by A and its $y-$intercept by B. What is the smallest possible area of triangle OAB?

10. The initial location of an object is $L\,(0)=8$. The picture shows the velocity function of this object. Find $L\,(10)$.



Answers

1. a) e^2 b) 1 c) undefined d) $\dfrac{1}{2}$ e) $\dfrac{5}{3}$

2. a) $x\sin x$ b) $-2\sin x\cos x=-\sin 2x$ c) $\sec^2 r$ d) $\dfrac{1}{x\left(x^2+1\right)}-\dfrac{2x\ln x}{\left(x^2+1\right)^2}$ e) $\dfrac{3x+2x^3}{\sqrt{3x^2+x^4+2}}$

   f) $\dfrac{-24}{(3x-5)^9}$ g) $\tan x$

3. a) $\dfrac{1}{2}x^2-\ln|x|+C$ b) $\dfrac{1}{3}x^3+\dfrac{2}{3}x\sqrt{x}+2\sqrt{x}-\dfrac{1}{x}+C$ c) $-\cos x+C$

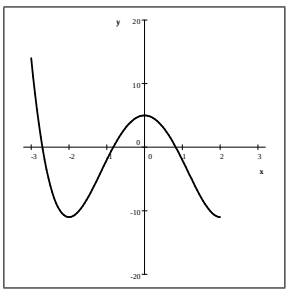
4. a) $f\,(x)=x^4-8x^2+5$ on $[-3,2]$

   relative maximum: $(0,5)$

   absolute maximum: $(-3,14)$

   relative minimum: $(-2,-11)$

   absolute minimum: $(-2,-11)$ and $(2,-11)$

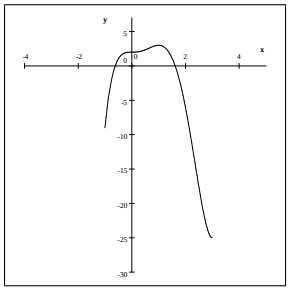
   b) $f\,(x)=x^5-5x^4+5x^3+2$ on $[-1,3]$

   relative maximum: $(1,3)$

   absolute maximum: $(1,3)$

   relative minimum: none

   absolute minimum: $(3,-25)$

5. a) $\dfrac{2}{3}\sqrt{21}$ b) 1 c) $\dfrac{\pi}{2}$ d) $\sqrt{5}$ 6. $v\,(0)=14\dfrac{\text{m}}{\text{s}}$ and $s\,(0)=80\,\text{m}$

7. horizontal sides: $2\sqrt{30}$ vertical sides: $\sqrt{30}$ 8. $\left(9-3\sqrt{3}\right)\,\text{cm}$ 9. 24 unit^2 when $m=-\dfrac{1}{3}$ 10. 14