Students must be able to correctly state the following theorems: Intermediate Value Theorem (both forms), Extreme Value Theorem, Rolle's Theorem, Mean Value Theorem, the second derivative test.

Students must be able to prove the following theorems:

- Differentiating functions using the definition (limit of the differential quotient)
- Prove that $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ and $\frac{d}{dx}(\ln x) = \frac{1}{x}$, $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(a^x) = a^x \cdot \ln a$, $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ and $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2+1}$
- If a function is differentiable at a number x, then it is continuous there.
- The product rule and quotient rule for derivatives, Rolle's Theorem, The Mean Value Theorem.
- (New!) If a sequence is increasing and bounded from above, then it has a limit.

Review Problems

1. Prove that if $f(x) = \tan^{-1} x$, then $f'(x) = \frac{1}{1+x^2}$
2. Compute each of the given limits. Show all steps.

   a) $\lim_{x\to 0} \frac{\sin 2x}{\tan 3x}$ c) $\lim_{x\to\infty} \frac{2^{3x+1}}{3^{2x-1}}$ e) $\lim_{x\to 1^+} \frac{x^2-3x+2}{x^2-1}$

   b) $\lim_{x\to\infty} \left(1-\frac{1}{x}\right)^x$ d) $\lim_{x\to 2} \frac{|6x-3|}{x-2}$ f) $\lim_{x\to -1^-} \frac{x^2-3x+2}{x^2-1}$
3. Differentiate each of the following.

   a) $f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$ b) $x^2 - y^2 = 2xy$ c) $\sin x = \tan^{-1} y$
4. Suppose that $f(x) = x^3$ on $[0,4]$. Let P be a uniform partition with n subintervals. Compute the left- and right Riemann sums if

   a) $n = 4$ b) $n = 10$ c) $n = 100$ d) in terms of n
5. Suppose that $f(x) = 3^x$ on $[0,1]$. Let P be a uniform partition with 5 subintervals. Compute the left- and right Riemann sums. Round your answer to three or more decimals.
6. Compute each of the given integrals.

   a) $\int \frac{2x-5}{x-1}dx$ b) $\int \frac{1}{\sqrt{1-x^2}}dx$ c) $\int 5^x dx$ d) $\int \frac{1}{1+x^2}dx$ e) $\int \frac{x^2}{1+x^2}dx$
7. The surface of a sphere is increasing at a constant rate of $5\pi \frac{\text{cm}^2}{\text{min}}$. At what rate is the radius growing when it is $20\,\text{cm}$?
8. a) A kite is $40\,\text{ft}$ high with $50\,\text{ft}$ cord out. If the kite moves horizontally at 5 feet per second directly away from the boy flying it, how fast is the cord being paid out?

   b) At the same time, at what rate is the slope of the cord changing?
9. Two cars are rented from the same rental agency. At noon one car starts to move with a speed of $30\frac{\text{mi}}{\text{h}}$. An hour later, the second car starts to move with a speed of $50\frac{\text{mi}}{\text{h}}$. The directions of the cars form a $60°$ angle. At what rate is the distance between the cars changing at 2 pm? (hint: use the law of cosines!) Present your answer as an approximation, accurate up to three or more decimals.

10. The kinetic energy of an object is $\frac{1}{2}mv^2$ where m is the mess and v is the velocity of the object. The mess of the object is $0.07\,\text{kg}$. Suppose that this object is falling with an acceleration of approximately $-10\frac{\text{m}}{\text{s}^2}$. At what rate is the object's kinetic energy changing at the time when its velocity is $200\frac{\text{m}}{\text{s}}$?

11. The base radius of a cylinder is increasing at a constant rate of $2\frac{\text{in}}{\text{min}}$, and its height is decreasing at a constant rate of $-5\frac{\text{in}}{\text{min}}$.
    a) At what rate is the cylinder's volume changing when the radius is 10 inches and the height is 50 inches long?
    b) At what rate is the cylinder's surface area changing when the radius is 10 inches and the height is 50 inches long?

12. We are designing a poster with a print area of $600\,\text{in}^2$. We plan to have a margin of 2 inches at the top and bottom, and 1 inch at the sides. What dimensions would result in the poster with the smallest possible area?

Answers

1. see handout 2. a) $\frac{2}{3}$ b) $\frac{1}{e}$ c) 0 d) undefined e) $-\frac{1}{2}$ f) ∞

3. a) $x^2\ln x$ b) $\frac{x-y}{x+y}$ c) $\left(y^2+1\right)\cos x$

4. a) $L = 36$ $R = 100$ b) $L = 51.84$ $R = 77.44$ c) $L = 62.7264$ $R = 65.2864$
   d) $L = \frac{64n^2 - 128n + 64}{n^2}$ $R = \frac{64n^2 + 128n + 64}{n^2}$ 5. 1.6278

6. a) $2x - 3\ln|x-1| + C$ b) $\sin^{-1}x + C$ c) $\frac{5^x}{\ln 5} + C$ d) $\tan^{-1}x + C$ e) $x - \tan^{-1}x + C$

7. a) $\frac{1}{32}\frac{\text{cm}}{\text{min}}$ 8. a) $3\frac{\text{ft}}{\text{s}}$ b) $m' = -\frac{2}{9}$ the slope is decreasing at a rate of $\frac{2}{9}$ 9. $36.819087\frac{\text{mi}}{\text{h}}$

10. $-140\frac{\text{kg}\,\text{m}^2}{\text{s}^3}$ 11. vertical sides: $20\sqrt{3}+4$ inches long horizontal sides: $10\sqrt{3}+2$ inches long

12. a) $1500\pi\frac{\text{in}^3}{\text{min}}$ b) $180\pi\frac{\text{in}^2}{\text{min}}$