Quiz 7 will cover the following material:

All material covered in Classes 1-9
All problems from Are You Ready For Calculus?
Differentiation 1 (Proofs)
Differentiation 1 (Practice)
Differentiating sinx and cosx (proofs)
Trigonometric Limits
Tangent Lines

The following Sample Quiz is intended to demonstrate the difficulty level of the questions on the quiz. It is not intended as a comprehensive review or list of the type of questions that can appear on the quiz.

## Sample Quiz 7

1. a) Compute the derivative of $f(x) = \cos x$ by evaluating the limit of the difference quotient.
   b) Differentiate $f(x) = \sqrt{5-2x}$ by computing the limit of the difference quotient.
2. Compute each of the given limits.

   a) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x}$ b) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$ c) $\lim\limits_{x\to 0} \dfrac{x}{\sin x + \sin 5x}$
3. Find the value of $m$ so that the given function is continuous on $\mathbb{R}$.
$$f(x) = \begin{cases} -x+15 & \text{if } x \le 3 \\ mx^2 - 2x & \text{if } x > 3 \end{cases}$$
4. Differentiate each of the following.

   a) $f(x) = -5x^3 + x^2 - 3x + 1$
   b) $g(x) = \sqrt{x} - \dfrac{1}{x} + \dfrac{1}{\sqrt{x}}$
   c) $r(\theta) = \sin^2\theta + \cos^2\theta$
   d) $y = \sqrt[3]{x^5} - \dfrac{1}{x^4} + e^2$
5. Find all values of $x$ for which the tangent line drawn to $f(x) = \sqrt[3]{x}$ is perpendicular to the line $y + 12x = 20$.
6. Find the point on the graph of $f(x) = -\dfrac{1}{4}x^2 + 5x + 1$ that is closest to the line segment $AB$ where $A(-4, 20)$ and $B(2, 32)$.

## Answers

1. a) Claim: $\dfrac{d}{dx}(\cos x) = -\sin x$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{\cos(x+h)} - \cos x}{h} = \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} = \\ &= \lim_{h\to 0} \frac{\cos x \cos h - \cos x}{h} + \lim_{h\to 0} \frac{-\sin x \sin h}{h} = \lim_{h\to 0} \frac{\cos x(\cos h - 1)}{h} + \lim_{h\to 0} \frac{-\sin x \sin h}{h} \\ &= \cos x \cdot \lim_{h\to 0} \frac{(\cos h - 1)}{h} - \sin x \cdot \lim_{h\to 0} \frac{\sin h}{h} = \cos x \cdot 0 - \sin x \cdot 1 = -\sin x \end{aligned}$$

b) Claim: $\frac{d}{dx}\left(\sqrt{5-2x}\right) = -\frac{1}{\sqrt{5-2x}}$

Proof:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{\sqrt{5-2(x+h)}-\sqrt{5-2x}}{h} = \lim_{h\to 0}\frac{\sqrt{5-2(x+h)}-\sqrt{5-2x}}{h}\cdot\frac{\sqrt{5-2(x+h)}+\sqrt{5-2x}}{\sqrt{5-2(x+h)}+\sqrt{5-2x}}\\
&= \lim_{h\to 0}\frac{(5-2x-2h)-(5-2x)}{h\left(\sqrt{5-2(x+h)}+\sqrt{5-2x}\right)} = \lim_{h\to 0}\frac{5-2x-2h-5+2x}{h\left(\sqrt{5-2(x+h)}+\sqrt{5-2x}\right)}\\
&= \lim_{h\to 0}\frac{-2h}{h\left(\sqrt{5-2(x+h)}+\sqrt{5-2x}\right)} = \lim_{h\to 0}\frac{-2}{\sqrt{5-2(x+h)}+\sqrt{5-2x}} = \frac{-2}{\sqrt{5-2x}+\sqrt{5-2x}}\\
&= \frac{-2}{2\sqrt{5-2x}} = \boxed{\frac{-1}{\sqrt{5-2x}} = \frac{-\sqrt{5-2x}}{5-2x}}
\end{aligned}$$

2. a) 3  b) $\frac{1}{2}$  c) $\frac{1}{6}$

hint for c): either try $\lim_{x\to 0}\frac{\sin x+\sin 5x}{x}$ or use the sum-product identities.

3. 2

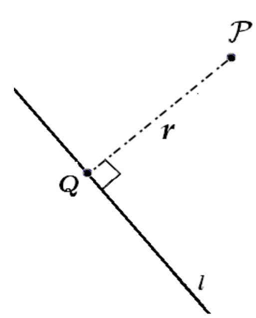
4. a) $f'(x) = -15x^2+2x-3$  b) $g'(x) = \frac{1}{2\sqrt{x}}+\frac{1}{x^2}-\frac{1}{2x\sqrt{x}} = \frac{\sqrt{x}}{2x}+\frac{1}{x^2}-\frac{\sqrt{x}}{2x^2}$  c) $r'(\theta)=0$

d) $\frac{d}{dx}\left(\sqrt[3]{x^5}-\frac{1}{x^4}+e^2\right) = \frac{5}{3}x^{2/3}+\frac{4}{x^5} = \frac{5\sqrt[3]{x^2}}{3}+\frac{4}{x^5}$

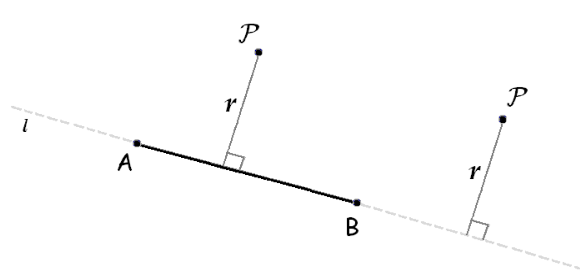
5. $x=\pm 8$  6. $(6,22)$

Solution for 6.

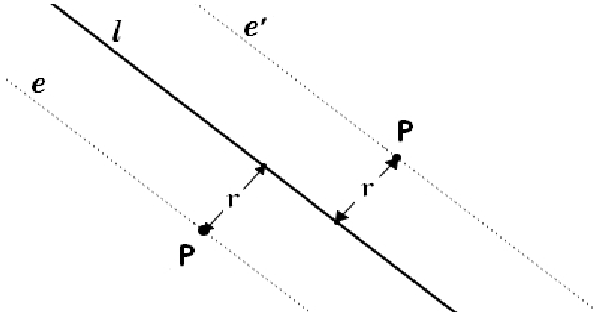
**Definition:** The **distance between a line $l$ and a point $P$** is the distance between $P$ and $Q$ where $Q$ is the intersection of line $l$ and the line passing through $P$ and perpendicular to $l$.




**Definition:** The **distance between a point $P$ and a line segment $AB$** is defined similarly, as the distance of point $P$ from the line $l$ determined by line segment $AB$.



Consequently, the set of all points in the plane equidistant to a line all lie on a pair of parallel lines.




The set of all points equidistant to a line segment lie on lines parallel to the line segment. Imagine we take a line parallel to our line segment and shift it toward the parabola. The closest point to line segment $AB$ will be the point that is on the tangent line. So, we need to find the tangent line that is parallel to line segment $AB$. The point of tangency is the point closest to line segment $AB$. We find the slope of line segment $AB$

$$m_{AB} = \frac{32-20}{2-(-4)} = \frac{12}{6} = 2$$

Thus we need to find $P$ on the graph of $f$ such that the tangent line drawn to the graph of $f$ at $P$ has slope 2. The derivative measures the slope of the tangent line.

$$x = ? \quad \text{so that } f'(x) = 2$$

$$\begin{aligned} f(x) &= -\frac{1}{4}x^2 + 5x + 1 \\ f'(x) &= -\frac{1}{2}x + 5 \\ 2 &= -\frac{1}{2}x + 5 \\ -3 &= -\frac{1}{2}x \\ 6 &= x \end{aligned}$$

Thus the closest point is $(6, f(6))$. Since $f(6) = 22$, the answer is $P(6, 22)$.