1. Simplify each of the following

   a) 6^{-2} c) $64^{-2/3}$ e) $8^{\log_2 5}$ g) $\log_3 4 \cdot \log_4 5 \cdot \log_5 6 - 1$ i) $e^{-2\ln A}$

   b) $32^{3/5}$ d) $\log_9 \sqrt{27}$ f) $3^{\log_9 10}$ h) $\log_2\left(4x^2\right) - 3\log_2\left(\dfrac{6}{x}\right) + \log_4\left(4x^6\right)$

2. Factor $3x^2 - 4x - 319$ by completing the square.

3. In case of each polynomial given, determine (by completing the square) whether it can be factored over the real numbers or not. (You do not have to actually factor.)

   a) $20x + 2x^2 + 44$ b) $20x - 5x^2 - 25$

4. Solve the equation $9x^2 - 12x = 11$ and check one of your solution using exact values.

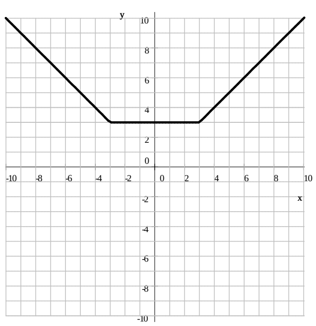
5. Find the equation for the inverse for each of the following functions given.

   a) $f(x) = 3x - 1$ c) $h(x) = 1 + 3e^{2x-7}$ e) $m(x) = \dfrac{x-1}{2x+7}$

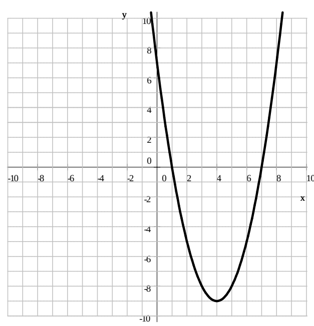
   b) $g(x) = \log_2(5x - 1)$ d) $p(x) = (x-2)^3$

6. In each case, the graph of a function is given. Sketch the graph of the inverse relation in the same coordinate system.

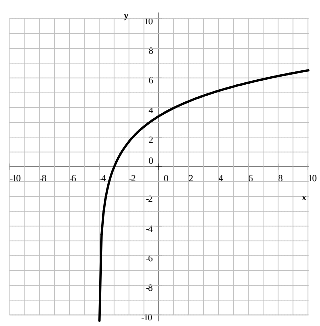
a)



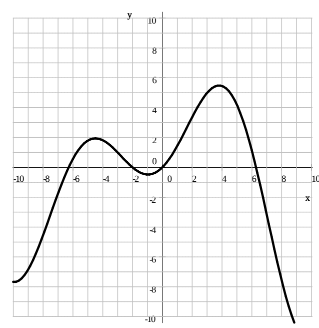
b)



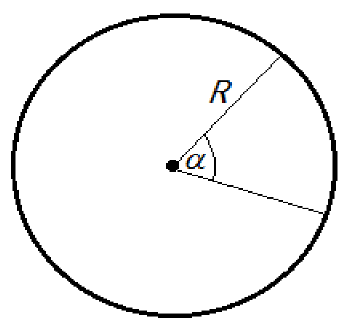
c)



d)



7. Suppose that α is a central angle (less than $360°$) in a circle with radius R.



   a) Express the length of the arc subtended by the central angle α in terms of α and R. Assume that α is measured in degrees.

   b) Express the area of the sector determined by the central angle α in terms of α and R. Assume that α is measured in degrees.

   c) Express the length of the arc subtended by the central angle α in terms of α and R. Assume that α is measured in radians.

   d) Express the area of the sector determined by the central angle α in terms of α and R. Assume that α is measured in radians.

8. Solve each of the following equations.

   a) $2x^3 = 6x$

   b) $2x^2 - 3x - 1 = 0$

   c) $\log_2(x+5) - \log_2(x-7) = -1$

   d) $\log_3(7-x) + \log_3(1-x) = 3$

   e) $\log_6(-8-x) + \log_6(8-x) = 2$

   f) $\dfrac{2x-1}{3} - \dfrac{x-1}{2} = x - 4$

9. Solve each of the following inequalities.

   a) $x^2 \geq 4x$ b) $8x + x^2 < 33$ c) $x^2 < -2x + 2$ d) $4x^2 \leq 4x - 1$ e) $x^2 - 6x > -10$

10. Find the domain for each of the following functions.

    a) $f(x) = \ln\left(x^2 - 10x + 29\right)$

    b) $g(x) = \dfrac{1}{\log_2(4 - x)}$

    c) $f(x) = \log_5\left(x^2 - 10x + 21\right)$

    d) $k(x) = \dfrac{1}{\log_5\left(x^2 - 10x + 21\right)}$

11. Find an equation for the curve that consists of points $P(x, y)$ with the following property: they are twice as far from point $A(2, -5)$ as from point $B(-1, 1)$.

12. a) Solve the formula $V = 2\pi r^3 + \dfrac{1}{2}\pi r^2 h$ for h.

    b) A right pyramid has a square base with sides 30 meters long. The pyramid is 24 meters tall. At what height is the perpendicular cross section a square with sides 10 meters?

13. Graph each of the following pairs of functions in the same coordinate system.

    a) $f(x) = 2^x$ and $g(x) = \log_2 x$ b) $f(x) = 2^x$ and $g(x) = \left(\dfrac{1}{2}\right)^x$

    c) $f(x) = \log_2 x$ and $g(x) = \log_{1/2} x$ d) $f(x) = \left(\dfrac{1}{2}\right)^x$ and $g(x) = \log_{1/2} x$

14. For each of the following functions given, give a complete analysis and sketch its graph.

    a) $f(x) = 9 - 3x^2 - 6x$ on $[-2, 3]$ b) $f(x) = \sqrt{x + 3}$ c) $f(x) = \log_3 x$ d) $f(x) = 0.7^x$

15. An object's height (measured in feet) is defined by $s(t) = 0.4t + 12$ where t is the time, measured in seconds.

    a) Find the location of the object at $t = 10$ seconds.

    b) Find the average velocity of the object between $t = 0$ and $t = 3$ seconds

    c) Find the average velocity of the object between $t = 5$ and $t = 10$ seconds

16. An object's height (measured in feet) is defined by $s(t) = t^3 - 12t$ where t is the time, measured in seconds.

    a) Find the location of the object at $t = 3$ seconds.

    b) Find the average velocity of the object between

       i) $t = 0$ and $t = 2$ seconds ii) $t = 1$ s and $t = 2$ s iii) $t = 1.5$ s and $t = 2$

17. Find the coordinates of all points where the graphs of $f(x) = x^2 - 2x - 26$ and $g(x) = 2x - 5$ intersect each other.

18. Compute the exact value of each of the following.

    a) $\sin(300°)$

    b) $\cos\left(-\dfrac{7\pi}{4}\right)$

    c) $\tan(900°)$

    d) $\sec(-300°)$

    e) $\cot\left(\dfrac{\pi}{6}\right)$

    f) $\sin^{-1}\left(\dfrac{1}{2}\right)$

    g) $\tan^{-1}(0)$

    h) $\cos^{-1}\left(-\dfrac{1}{2}\right)$

    i) $\sin^{-1}\left(-\dfrac{1}{2}\right)$

19. Solve each of the given equations.

    a) $\cos x + 2 = 1 + 2\sin^2 x$ b) $\tan^2 x + \tan x = 0$ c) $\sin x = 2\sin x \cos x$

20. Compute each of the following limits.

a) $\lim_{x\to-\infty}\left(-2x^5+8x^2\right)$

b) $\lim_{x\to\infty}\left(-2x^5+8x^2\right)$

c) $\lim_{x\to-\infty}\left(-2x^5+8x^6\right)$

d) $\lim_{x\to\infty}\left(-2x^5+8x^6\right)$

e) $\lim_{x\to-\infty}\dfrac{3x^2-1}{5x^2-3x+2}$

f) $\lim_{x\to-\infty}\dfrac{100x-1}{5x^2-3x+2}$

g) $\lim_{x\to-\infty}\log_2 x$

h) $\lim_{x\to\infty}\dfrac{2x^2+3x+1}{3x^2-5x+2}$

i) $\lim_{x\to\infty}\dfrac{-x^3+2x+1}{x-3}$

j) $\lim_{x\to-\infty}2^x$

k) $\lim_{x\to\infty}\left(\log_2\left(x^2-5x+17\right)\right)$

l) $\lim_{x\to\infty}\dfrac{12+\log_7 3x}{15+\log_7 x}$

m) $\lim_{x\to\infty}\dfrac{2^{x+5}}{4^{x-1}}$

n) $\lim_{x\to\infty}\dfrac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

o) $\lim_{x\to\infty}x\left(\dfrac{1}{3}-\dfrac{1}{3-\frac{1}{x}}\right)$

p) $\lim_{x\to\infty}\dfrac{\sqrt{4-\frac{1}{x}}-2}{\frac{1}{x}}$

q) $\lim_{x\to-\infty}\dfrac{\cos x-2}{x^3+1}$

r) $\lim_{x\to\infty}\tan x$

s) $\lim_{x\to\infty}\dfrac{\ln 2x}{\ln 3x}$

t) $\lim_{\theta\to\infty}\left(\sin^2\theta+\cos^2\theta\right)$

u) $\lim_{x\to\infty}\dfrac{\sqrt{2x^2-1}}{x}$

v) $\lim_{x\to-\infty}\tan^{-1}x$

21. Based on the graph of a function f shown on the picture, determine each of the following limits.

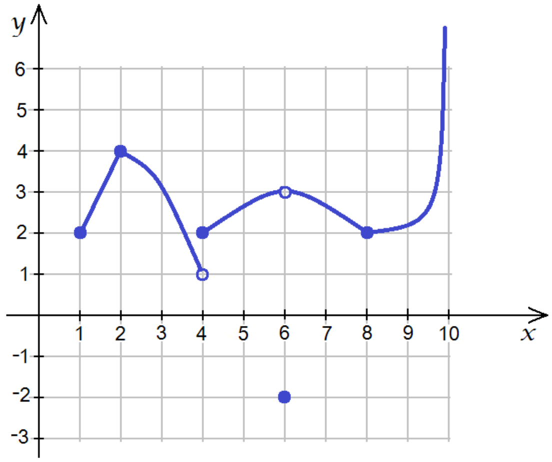
a) $\lim_{x\to1^-}f(x)$ b) $\lim_{x\to1^+}f(x)$ c) $\lim_{x\to1}f(x)$

d) $\lim_{x\to2^-}f(x)$ e) $\lim_{x\to2^+}f(x)$ f) $\lim_{x\to2}f(x)$

g) $\lim_{x\to4^-}f(x)$ h) $\lim_{x\to4^+}f(x)$ i) $\lim_{x\to4}f(x)$

j) $\lim_{x\to6^-}f(x)$ k) $\lim_{x\to6^+}f(x)$ l) $\lim_{x\to6}f(x)$

m) $\lim_{x\to10^-}f(x)$ n) $\lim_{x\to10^+}f(x)$ o) $\lim_{x\to10}f(x)$



22. Let $f(x)=\dfrac{x^2-9}{2x^2-8x+6}$. Compute each of the following limits.

a) $\lim_{x\to-\infty}f(x)$ b) $\lim_{x\to-3^-}f(x)$

c) $\lim_{x\to-3^+}f(x)$ d) $\lim_{x\to-3}f(x)$

e) $\lim_{x\to1^-}f(x)$ f) $\lim_{x\to1^+}f(x)$

g) $\lim_{x\to1}f(x)$ h) $\lim_{x\to3^-}f(x)$

i) $\lim_{x\to3^+}f(x)$ j) $\lim_{x\to3}f(x)$

k) $\lim_{x\to\infty}f(x)$

23. Based on your results in the previous problem, sketch the graph of $f(x) = \dfrac{x^2-9}{2x^2-8x+6}$. (Plot a few additional points if needed.)

24. Compute each of the following limits.

a) $\displaystyle\lim_{x\to 2} \frac{2-x}{\frac{1}{2}-\frac{1}{x}}$

b) $\displaystyle\lim_{x\to 1^-} \frac{x^2-1}{2-\sqrt{3+x}}$

c) $\displaystyle\lim_{x\to 3} \frac{|2x-4|}{x-8}$

d) $\displaystyle\lim_{x\to 0^-} \frac{|x|}{x}$

e) $\displaystyle\lim_{x\to 0} \frac{|x|}{x}$

f) $\displaystyle\lim_{x\to 1^+} \frac{\ln x}{\sqrt{1-x}}$

g) $\displaystyle\lim_{x\to 0} \frac{\sqrt{5+x}-\sqrt{5}}{x}$

h) $\displaystyle\lim_{x\to \sqrt{3}} \tan^{-1} x$

i) $\displaystyle\lim_{x\to \pi} \frac{\cos x}{\sin x}$

j) $\displaystyle\lim_{x\to \pi^+} \frac{\cos x}{\sin x}$

k) $\displaystyle\lim_{x\to \pi/4} \tan x$

l) $\displaystyle\lim_{a\to 0} \frac{\sqrt{5a+4}-2}{a}$

m) $\displaystyle\lim_{m\to 1^-} \frac{m^2-1}{m^4-1}$

n) $\displaystyle\lim_{x\to 0} \frac{1+x+\sin x}{\cos x}$

o) $\displaystyle\lim_{x\to \pi/3} \left(\tan^2 x - 1\right)$

p) $\displaystyle\lim_{h\to 0} \frac{\sqrt{h^2+100}-10}{h^2}$

q) $\displaystyle\lim_{x\to 0^+} \ln x$

25. Find the derivative of each of the following functions by evaluating the difference quotient.

a) $f(x) = 3x-8$ b) $f(x) = \sqrt{3x}$ c) $f(x) = \dfrac{1}{5x-1}$

26. Find an equation for the tangent line drawn to the graph of $f(x) = \sqrt{3x}$ at $x = 4$.

27. Find all values of x forn which the tangent line drawn to the graph of $f(x) = \dfrac{1}{x}$ at x has a slope of -2.

Answers

1. a) $\dfrac{1}{36}$ b) 8 c) $\dfrac{1}{16}$ d) $\dfrac{3}{4}$ e) 125 f) $\sqrt{10}$ g) $\log_3 2 = \dfrac{\ln 2}{\ln 3}$ h) $\log_2\left(\dfrac{x^8}{27}\right)$ i) $\dfrac{1}{A^2}$

2. $3\left(x+\dfrac{29}{3}\right)(x-11) = (3x+29)(x-11)$ 3. a) can be factored b) cannot be factored

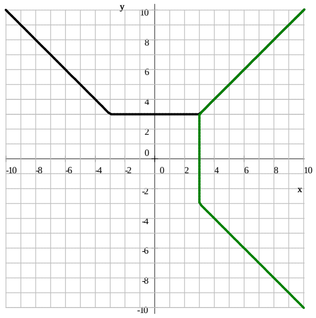
4. $\dfrac{2\pm\sqrt{15}}{3}$ Check: if $x = \dfrac{2-\sqrt{15}}{3}$, then

$$\begin{aligned} \text{LHS} &= 9x^2-12x = 9\left(\frac{2-\sqrt{15}}{3}\right)^2 - 12\left(\frac{2-\sqrt{15}}{3}\right) = 9\cdot\frac{\left(2-\sqrt{15}\right)^2}{9} - 12\cdot\frac{2-\sqrt{15}}{3} \\ &= \left(2-\sqrt{15}\right)^2 - 4\left(2-\sqrt{15}\right) = 4+15-4\sqrt{15}-8+4\sqrt{15} = 19-8 = 11 = \text{RHS} \end{aligned}$$

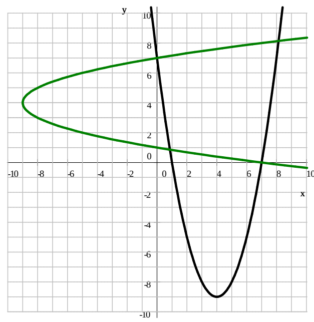
5. a) $f^{-1}(x) = \dfrac{1}{3}x+\dfrac{1}{3}$ b) $g^{-1}(x) = \dfrac{1}{5}(2^x+1)$ c) $h^{-1}(x) = \dfrac{1}{2}\ln\left(\dfrac{x-1}{3}\right)+\dfrac{7}{2}$

d) $p^{-1}(x) = \sqrt[3]{x}+2$ e) $m^{-1}(x) = \dfrac{7x+1}{-2x+1}$

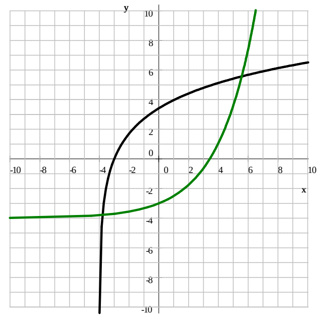
6. a)



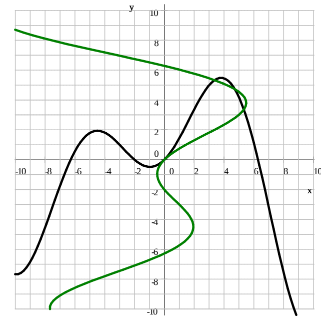
b)



c)



d)



7. a) $s = \dfrac{2\pi R\alpha}{360°}$ b) $A = \dfrac{\pi R^2 \alpha}{360°}$ c) $s = R\alpha$ d) $A = \dfrac{1}{2}R^2\alpha$

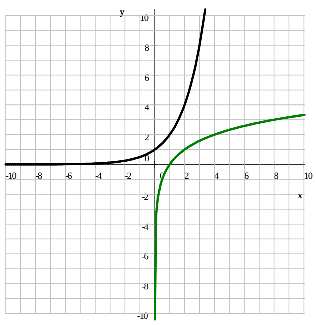
8. a) $0, -\sqrt{3}, \sqrt{3}$ b) $\dfrac{3 \pm \sqrt{17}}{4}$ c) no solution d) -2 e) -10 f) 5

9. a) $(-\infty, 0] \cup [4, \infty)$ b) $(-11, 3)$ c) $\left(-\sqrt{3}-1, \sqrt{3}-1\right)$ d) $x = \dfrac{1}{2}$ e) $\mathbb{R}$

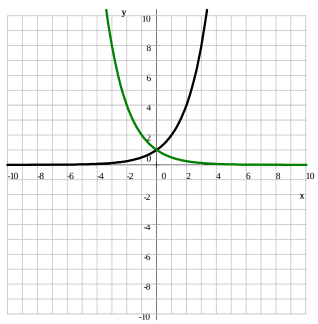
10. a) $\mathbb{R}$ b) $x < 4$ but $x \neq 3$ c) $x < 3$ or $x > 7$ d) $x < 3$ but $x \neq 5 - \sqrt{5}$ or $x > 7$ but $x \neq 5 + \sqrt{5}$

11. $(x+2)^2 + (y-3)^2 = 20$ 12. a) $h = \dfrac{2\left(V - 2\pi r^3\right)}{\pi r^2}$ or $h = \dfrac{2V}{\pi r^2} - 4r$ b) At a height of 16 meters

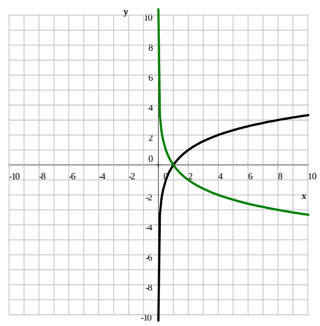
13. a)



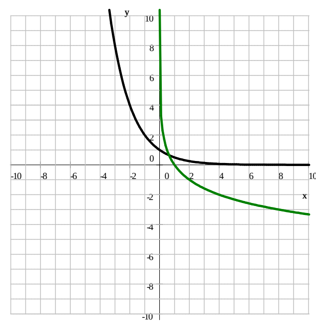
b)



c)



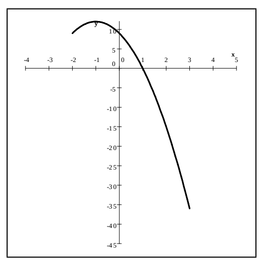
d)



14. a) $f(x) = 9 - 3x^2 - 6x$ on $[-2, 3]$

domain: $[-2, 3]$
range: $[-36, 12]$
$x-$intercept: $(1, 0)$
$y-$ intercept: $(0, 9)$
maximum: $(-1, 12)$
minimum: $(3, -36)$

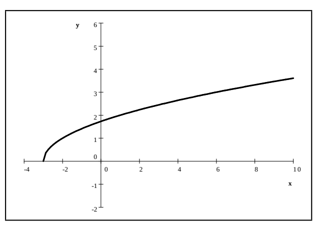
one-to-one: no
end-behavior: none
i.e. $\lim_{x \to \pm\infty} f(x) =$ undefined



b) $f(x) = \sqrt{x+3}$

domain: $[-3, \infty)$
range: $[0, \infty)$
$x-$intercept: $(-3, 0)$
$y-$ intercept: $(0, \sqrt{3})$
maximum: none
minimum: $(-3, 0)$

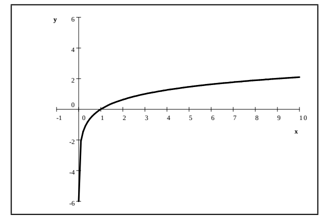
one-to-one: yes
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
and $\lim_{x \to \infty} f(x) = \infty$



c) $f(x) = \log_3 x$

domain: $(0, \infty)$
range: $\mathbb{R}$
$x-$intercept: $(1, 0)$
$y-$ intercept: none
maximum: none
minimum: none

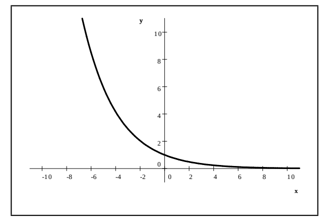
one-to-one: yes
end-behavior:
$\lim\limits_{x\to-\infty} f(x) =$ undefined
and $\lim\limits_{x\to\infty} f(x) = \infty$



d) $f(x) = 0.7^x$

domain: $\mathbb{R}$
range: $(0, \infty)$
$x-$intercept: none
$y-$ intercept: $(0, 1)$
maximum: none
minimum: none

one-to-one: yes
end-behavior:
$\lim\limits_{x\to-\infty} f(x) = \infty$
and $\lim\limits_{x\to\infty} f(x) = 0$



15. a) $16\,\text{ft}$ b) $0.4\frac{\text{ft}}{\text{s}}$ c) $0.4\frac{\text{ft}}{\text{s}}$ 16. a) $-9\,\text{ft}$ b) i) $-8\frac{\text{ft}}{\text{s}}$ ii) $-5\frac{\text{ft}}{\text{s}}$ iii) $-2.75\frac{\text{ft}}{\text{s}}$

17. $(7, 9)$ and $(-3, -11)$ 18. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) 0 d) 2 e) $\sqrt{3}$ f) $\frac{\pi}{6}$ g) 0 h) $\frac{2\pi}{3}$ i) $-\frac{\pi}{6}$

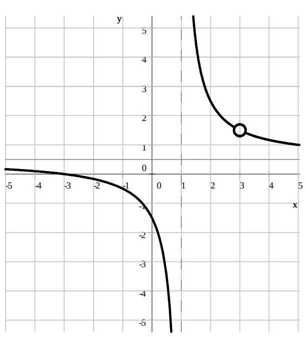
19. a) $\pi + 2k\pi,\ \frac{\pi}{3} + 2k\pi,\ -\frac{\pi}{3} + 2k\pi$ where k is an integer
b) $k\pi,\ -\frac{\pi}{4} + k\pi$ where k is an integer c) $k\pi \quad \pm\frac{\pi}{3} + 2k\pi$ where k is an integer

20. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0 k) ∞
l) 1 m) 0 n) 3 o) $-\frac{1}{9}$ p) $-\frac{1}{4}$ q) 0 r) undefined s) 1 t) 1 u) $\sqrt{2}$ v) $-\frac{\pi}{2}$

21. a) undefined b) 2 c) undefined d) 4 e) 4 f) 4 g) 1 h) 2 i) undefined
j) 3 k) 3 l) 3 m) ∞ n) undefined o) undefined

22. a) $\frac{1}{2}$ b) 0 c) 0 d) 0 e) $-\infty$ f) ∞ g) undefined h) $\frac{3}{2}$ i) $\frac{3}{2}$ j) $\frac{3}{2}$ k) $\frac{1}{2}$

23.



24. a) -4 b) -8 c) $-\frac{2}{5}$ d) -1 e) undefined f) undefined g) $\frac{\sqrt{5}}{10}$
h) $\frac{\pi}{3}$ i) undefined j) ∞ k) 1 l) $\frac{5}{4}$ m) $\frac{1}{2}$ n) 1 o) 2 p) $\frac{1}{20}$ q) $-\infty$

25. a) $f'(x) = \lim\limits_{h\to0} \frac{3(x+h) - 8 - (3x - 8)}{h} = \lim\limits_{h\to0} \frac{3x + 3h - 8 - 3x + 8}{h} = \lim\limits_{h\to0} \frac{3\cancel{h}}{\cancel{h}} = \lim\limits_{h\to0} 3 = 3$

b) $f'(x) = \lim_{h\to 0} \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h} = \lim_{h\to 0} \frac{\sqrt{3x+3h} - \sqrt{3x}}{h} \cdot \frac{\sqrt{3x+3h} + \sqrt{3x}}{\sqrt{3x+3h} + \sqrt{3x}} = \lim_{h\to 0} \frac{3x+3h-3x}{h\left(\sqrt{3x+3h} + \sqrt{3x}\right)}$

$= \lim_{h\to 0} \frac{3\not{h}}{\not{h}\left(\sqrt{3x+3h} + \sqrt{3x}\right)} = \frac{3}{\sqrt{3x} + \sqrt{3x}} = \frac{3}{2\sqrt{3x}}$

c) $\lim_{h\to 0} \frac{\frac{1}{5(x+h)-1} - \frac{1}{5x-1}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{5x+5h-1} - \frac{1}{5x-1}\right)$

$= \lim_{h\to 0} \frac{1}{h}\left(\frac{5x-1}{(5x+5h-1)(5x-1)} - \frac{5x+5h-1}{(5x+5h-1)(5x-1)}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{5x-1-(5x+5h-1)}{(5x+5h-1)(5x-1)}\right)$

$= \lim_{h\to 0} \frac{1}{h}\left(\frac{5x-1-5x-5h+1}{(5x+5h-1)(5x-1)}\right) = \lim_{h\to 0} \frac{1}{\not{h}}\left(\frac{-5\not{h}}{(5x+5h-1)(5x-1)}\right) = \lim_{h\to 0} \frac{-5}{(5x+5h-1)(5x-1)}$

$= \frac{-5}{(5x-1)(5x-1)} = \frac{-5}{(5x-1)^2}$

26. $y - \sqrt{12} = \frac{\sqrt{3}}{4}(x-4)$ 27. $\pm\frac{\sqrt{2}}{2}$