

Quiz 3 will cover the following material: (all handouts posted on the web site so far)

1. All material for Quizzes 1 and 2
2. Differentiate any function, including logarithmic, exponential, and inverse trigonometric functions.
3. Apply the fundamental theorem to compute definite integrals and differentiate functions defined using definite integrals.
4. Graph and state the basic properties of trigonometric functions, including inverse functions.
5. Integrate using substitution and integration by parts.

Sample Quiz 3

1. Graph $f(x) = \tan^{-1} x$ and state its properties.

2. Prove that $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{x^2 + 1}$

3. Differentiate each of the following:

- a) $f(x) = \cot x + \sec x$

- b) $m(x) = e^{\cos 5x}$

- c) $M(a) = \tan^{-1}(a^4)$

- d) $f(x) = \int_{x^4}^0 \sqrt{t^2 + 1} dt$

- e) $g(x) = \frac{d}{dx} \left(\int_0^{x^3} \frac{1}{\sqrt{1-t^2}} dt \right)^2$

- f) $f(x) = 3^{-2x+1}$

4. Compute each of the following integrals.

- a) $\int \frac{x}{x^2 + 3} dx$

- b) $\int \frac{1}{x^2 + 3} dx$

- c) $\int \tan^{-1} x dx$

- d) $\int e^x \cos x dx$

- e) $\int \csc x dx$

- f) $\int_1^e x^3 \ln x dx$

- g) $\int \tan^3 x dx$

- h) $\int_0^{\pi/4} \tan^3 x dx$

- i) $\int_1^8 \frac{1}{\sqrt{3x+1}} dx$

- j) $\int \sin(5x) \cos(11x) dx$

- k) $\int \cos^3 x dx$

- l) $\int \cos^4 x dx$

- m) $\int \cos^5 x dx$

- n) $\int e^x \cos x dx$

Answers

1. see handout

2. see handout

3. a) $f'(x) = -\cot^2 x - 1 + \sec x \tan x$ b) $m'(x) = -5(\sin 5x) e^{\cos 5x}$ c) $M'(a) = \frac{4a^3}{a^8 + 1}$

d) $f'(x) = -4x^3 \sqrt{x^8 + 1}$ e) $\frac{6x^2 \arcsin(x^3)}{\sqrt{1 - x^6}}$ f) $f'(x) = -2 \cdot (\ln 3) \cdot 3^{-2x+1}$

4. a) $\frac{1}{2} \ln(x^2 + 3) + C$ b) $\frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C$ c) $x \tan^{-1} x - \frac{1}{2} \ln(x^2 + 1) + C$

d) $\frac{1}{2} e^x (\sin x + \cos x) + C$ e) $-\ln|\csc x + \cot x| + C$ f) $\frac{3}{16} e^4 + \frac{1}{16}$

g) $\frac{1}{2} \tan^2 x + \ln|\cos x| + C$ h) $\frac{1}{2} - \frac{1}{2} \ln 2$ i) 2 j) $\frac{1}{12} \cos 6x - \frac{1}{32} \cos 16x + C$

k) $\sin x - \frac{\sin^3 x}{3} + C$ l) $\frac{3}{8} x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$

m) $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$ n) $\frac{1}{2} e^x (\sin x + \cos x) + C$