1. Simplify each of the following expressions.

a) $27^{2/3}$

b) $-27^{2/3}$

c) $(-27)^{2/3}$

d) $-27^{-2/3}$

e) $-8^{-5/3}$

f) $\sqrt[3]{x^{21}}$

g) $\sqrt[6]{x^{10}}$

h) $\dfrac{x^3y^0\left(-y^2xy^{-2}x^{-2}\right)^{-3}}{(2x)^{-2}\,yx^0}$

i) $x^{2/3}\cdot x^{-5/6}$

j) $\left(\sqrt[3]{x}\right)^2\left(\dfrac{1}{\sqrt[6]{x}}\right)^5$

k) $\left(\dfrac{a^{1/3}a^{1/2}}{a^{1/6}}\right)^{1/2}$

l) $\dfrac{x}{\sqrt{x}}$

m) $2^{-1}-5^{-1}$

n) $\dfrac{x^{-1}-y^{-1}}{x^{-2}+y^{-2}}$

o) $\left(\dfrac{1-x^{-1/3}}{3x^{2/3}}\right)^0$

2. Simplify each of the following expressions.

a) $\dfrac{\sqrt{5}-1}{-\sqrt{5}+1}$

b) $\dfrac{x^2-5x}{x^2-2x-15}\cdot\dfrac{x^2-9}{x^2-3x}$

c) $\dfrac{2am+2an-bm-bn}{b-2a+6ax-3bx}$

d) $\dfrac{1}{2-\dfrac{1}{x-4}}$

e) $\dfrac{2ax-3b-3a+2bx}{a^2-b^2}\div\dfrac{2x^2-x-3}{3b-3a+ax-bx}$

f) $\dfrac{2}{x^2-1}+\dfrac{1}{x+1}$

g) $\dfrac{2x-1}{x-3}-\dfrac{x+x^2+3}{x^2-3x}$

3. Simplify each of the following. If the denominator is irrational, rationalize it.

a) $\left(7-3\sqrt{2}\right)^2$

b) $\left(2-\sqrt{5}\right)^3\left(2+\sqrt{5}\right)^3$

c) $\dfrac{2}{\sqrt{29}+5}$

d) $\dfrac{4x-9}{2\sqrt{x}-3}$

e) $3\sqrt{8}+2\sqrt{18}-\sqrt{50}$

f) $\dfrac{-6+\sqrt{24}}{-10}$

4. Completely factor each of the following binomials over the real numbers. If an expression cannot be factored, state so.

a) x^2+x b) x^3-x c) x^4+x^3 d) x^5+x^3 e) x^6-x^4 f) x^7-x^3

5. Completely factor each of the following trinomials over the real numbers. If an expression cannot be factored, state so.

a) x^2-6x+5

b) x^2-6x+9

c) $x^2-6x+13$

d) x^2-6x+7

e) $3x^2-14x-5$

f) x^2-x-1

6. Solve each of the following system of equations.

a) $\begin{cases} 2x-5y=-6 \\ -x+\dfrac{1}{2}y=3 \end{cases}$

b) $\begin{cases} x-3y=6 \\ y=\dfrac{1}{3}x-2 \end{cases}$

c) $\begin{cases} x+y=3 \\ x=-y-1 \end{cases}$

d) $\begin{cases} 3x+5y=-20 \\ \dfrac{1}{3}x-\dfrac{1}{2}y=2 \end{cases}$

e) $\begin{cases} 3(x-5)-4(2y-1)=2x-5y \\ x+2-(y-5)=2(y-8) \end{cases}$

f) $\begin{cases} (x+5)^2-(y-5)^2=(x+y)(x-y)+10 \\ x+y=1 \end{cases}$

g) $\begin{cases} \dfrac{1}{2}(x+3)+\dfrac{1}{3}(y-2)=x-4 \\ \dfrac{1}{6}(x-1)-\dfrac{1}{2}(y+8)=y+3 \end{cases}$

7. Solve each of the following compound inequalities.

   a) $-\frac{1}{3}x+1<-8$ and $(x+1)^2-(x-1)^2\le -16$

   b) $-\frac{1}{3}x+1<-8$ or $(x+1)^2-(x-1)^2\le -16$

   c) $(2x-5)^2>4x^2-15x$ and $\frac{1}{3}x+1>\frac{5}{6}$

   d) $(2x-5)^2>4x^2-15x$ or $\frac{1}{3}x+1>\frac{5}{6}$

8. Solve each of the following equations over the real numbers. Make sure to check your solutions.

   a) $\frac{5-4x}{3}-\frac{2x-7}{5}=-2x+2$

   b) $x^3-2x^2-35x=0$

   c) $3|x+3|-5=10$

   d) $2(x-3)-\frac{x}{2}=\frac{3}{2}(x-4)$

   e) $2x^2-32x=0$

   f) $4x+x^3=6x^2$

   g) $x^2+134=22x$

   h) $3\left|\frac{1}{2}x-5\right|+1=-8$

   i) $5-(2-x)(x+3)=(x-2)^2$

9. Solve each of the following absolute value equations.

   a) $|2x+1|=-7$

   b) $|2x+1|=7$

   c) $|2x+1|=|x-5|$

   d) $|2x+1|=x-5$

   e) $3\left|\frac{1}{2}x+2\right|-1=20$

   f) $12-|3x-1|=8$

10. Graph each of the following functions. a) $f(x)=x^2$ b) $g(x)=x^2-4$ c) $h(x)=(x-4)^2$

11. Graph the parabola $y=-2x^2-4x+6$. State the coordinates of at least five points, including vertex and intercepts.

12. Find both coordinates of the vertex of each of the following parabolas and state whether it is a maximum or a minimum.

    a) $y=2x^2-16x+30$ b) $y=-3x^2-6x-15$ c) $y=\frac{1}{2}x^2-2x+7$

13. We are standing on the top of a 960 feet tall building and launch a small object upward. The object's vertical position, measured in feet, after t seconds is

$$h(t)=-16t^2+64t+960$$

    a) What is the highest point that the object reaches?

    b) How much did the object travel between being launched and hitting the ground?

14. Suppose that $f(x)$ is a function given by $f(x)=-x^2+10x-1$. Find the value of each of the folowing.

    a) $f(0)$

    b) $f(3)$

    c) $f(-3)$

    d) $\frac{f(-3)}{f(3)}$

    e) $f(-1)$

    f) $f\left(\sqrt{2}\right)$

    g) $f\left(5-\sqrt{2}\right)$

    h) $f(f(1))$

    i) $f(f(10))$

    j) Find all values of x with $f(x)=-1$

    k) Find all values of x with $f(x)=0$

15. Re-write each of the following decimals as a fraction of two integers. You do NOT have to bring the fraction to lowest terms.

    a) $0.65\overline{2}=0.652\,222\,222\ldots.$ b) $0.8\overline{79}=0.879\,797\,979\,797\,9\ldots$ c) $0.99\overline{2016}=0.992\,016\,201\,620\,16\ldots..$

16. Compute the sum $91+103+115+\ldots+715$

17. Find the exact value of the length of the main diagonal in a rectangular prism with edges 2 ft, 3 ft, and 5 ft long.

18. Find an equation for each of the following lines.

a) the line with slope $\frac{3}{4}$ and passing through the point $P(-12, 7)$

b) the line passing through the points $A(3, -1)$ and $B(-5, 7)$

c) the line passing through the point $P(6, 2)$ and is parallel to the line $3x - 2y = 5$.

d) the line passing through the points $N(-3, 4)$ and $M(-3, 7)$

e) the line passing through the point $P(6, 2)$ and is perpendicular to the line $3x - 2y = 5$.

19. Word Problems.

a) The difference between two integers is 26. Their product is 1767. Find these numbers.

b) How many gallons of 12% solution must be mixed with 6 gallons of 20% solution to obtain a solution that is 15%?

c) How many gallons of each of a 7% solution and a 27% solution should be mixed if we wanted to obtain 60 gallons of a 10% solution?

d) A bicycle leaves Boston, heading West at $5\frac{\text{mi}}{\text{h}}$. Exactly 24 hours later, a second bicycle leaves Boston, heading West at $13\frac{\text{mi}}{\text{h}}$. How long will to take for the second bicycle to overtake the first bicycle?

e) The population of a town has decreased from 75000 to 65250. What percent of a change does this represent?

f) The budget increased by 15%. If the new budget is 1811 250, how much was the old budget?

g) Town A and town B are located at a distance of 100 miles from each other. A jogger starts in town A and jogs toward town B. At the same time, a bicycle starts in town B and travels toward town A. The bicycle is moving $9\frac{\text{mi}}{\text{h}}$ faster than the jogger. Find the speeds of both the bycicle and the jogger if we know that they met exactly 4 hours after they started.

h) Leah headed South with a velocity of 11 miles per hour. 21 hours later Caitlyn followed her, with a velocity of 18 miles per hour. How long until Caitlyn catches up with Leah?

i) The sum of five times a number and -10 is 8 less than six times the sum of 7 and the opposite of the number. Find this number.

j) The dog was chasing the cat who had a 40 feet head start. The velocity of the dog was 9 feet per second while that of the cat was only 4 feet per second. How long until the dog catches up with the cat?

k) A plane leaves an airport and flies south at 800 miles per hours. Later, a second plane leaves the same airport and flies south at 1120 miles per hour. If the second plane overtakes the first one in 10 hours, how much earlier did the first plane leave?

l) We have invested \$8000 into two bank accounts: one earns 6% interest, the other one earns 9% interest. How much money did we invest into each account if the combined interest was \$660?

m) The digits in a two-digit number add up to 10. If we interchange the digits in the number, we obtain a new number that is 54 greater than the original number. Find the original number.

n) The ten's digit in a two-digit number is 4 greater than the one's digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 154. Find this number.

o) The hypotenuse of a right triangle is 26 cm. The difference between the other two sides is 14 cm. Find the missing sides.

p) Lisa took 5 exams. The first 4 received scores of 72, 93, 86, and 82. How much did she score on the fifth exam if her average score is 74 points?

q) A number is exactly one greater than its own reciprocal. Find this number.

r) The base of a right triangle is 8 units longer than its height. Find the base if we know that the area of the triangle is 120 unit^2.

s) Today we had 120 students visiting the museum as part of a field trip. Due to a soccer game, the girls team needed to leave earlier. 25% of the attending female students left early, causing a 15% decrease in the total attendance. How many of the attending students were male?

t*) Last week the shelter had lots of adoptable pets, all cats and dogs. On the great adoption day, three-fourth of the cats and one-sixth of the dogs were adopted. If these lucky pets represented half of the shelter's population, then what was the ration of cats to dogs?

Answers

1. a) 9 b) -9 c) undefined d) $-\frac{1}{9}$ e) $-\frac{1}{32}$ f) x^7 g) $\sqrt[3]{x^5}$ h) $\frac{-4x^8}{y}$ i) $x^{-1/6}$
   j) $\frac{1}{\sqrt[6]{x}}$ k) $\sqrt[3]{a}$ l) $\sqrt{x}$ m) $\frac{3}{10}$ n) $\frac{xy\,(y-x)}{x^2+y^2}=\frac{xy^2-x^2y}{x^2+y^2}$ o) 1

2. a) -1 b) 1 c) $\frac{m+n}{3x-1}$ d) $\frac{x-4}{2x-9}$ e) $\frac{x-3}{x+1}$ f) $\frac{1}{x-1}$ g) $\frac{x+1}{x}$

3. a) $67-42\sqrt{2}$ b) -1 c) $\frac{\sqrt{29}-5}{2}$ d) $2\sqrt{x}+3$ e) $7\sqrt{2}$ f) $\frac{3-\sqrt{6}}{5}$

4. a) $x\,(x+1)$ b) $x\,(x-1)\,(x+1)$ c) $x^3\,(x+1)$ d) $x^3\left(x^2+1\right)$ e) $x^4\,(x-1)\,(x+1)$ f) $x^3\left(x^2+1\right)(x-1)\,(x+1)$

5. a) $(x-1)\,(x-5)$ b) $(x-3)^2$ c) can not be factored d) $\left(x-3-\sqrt{2}\right)\left(x-3+\sqrt{2}\right)$
   e) $(3x+1)\,(x-5)$ f) $\left(x-\frac{1+\sqrt{5}}{2}\right)\left(x-\frac{1-\sqrt{5}}{2}\right)$

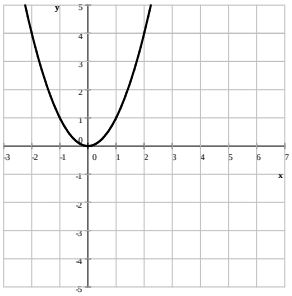
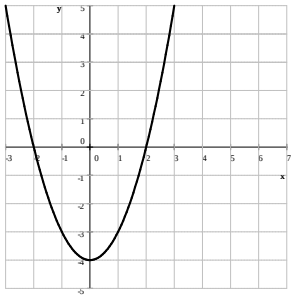
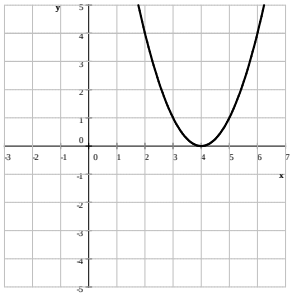
6. a) $(-3,0)$ b) this system is dependent, there are infinitely many solutions as follows: x can be any number, and then $y=\frac{1}{3}x-2$. Notation: $\left\{\left(x,\frac{1}{3}x-2\right)\text{ where }x\in\mathbb{R}\right\}$
   c) this sytem is inconsistent, there is no solution d) $(0,-4)$ e) this sytem is inconsistent, there is no solution
   f) this system is dependent, there are infinitely many solutions as follows: x can be any number, and then $y=-x+1$. Notation: $\{(x,-x+1)\text{ where }x\in\mathbb{R}\}$ g) $(7,-4)$

7. a) no solution b) $(-\infty,-4]\cup(27,\infty)$ c) $\left(-\frac{1}{2},5\right)$ d) $\mathbb{R}$

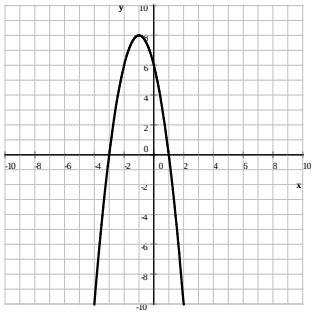
8. a) -4 b) $7,0,-5$ c) $-8,2$ d) identity, all numbers are solution e) $0,16$ f) $0,3-\sqrt{5},3+\sqrt{5}$
   g) no real solution h) no real solution i) 1

9. a) no solution b) $3,-4$ c) $-6,\frac{4}{3}$ d) no solution e) $10,-18$ f) $-1,\frac{5}{3}$

10. a) $f(x) = x^2$ b) $g(x) = x^2 - 4$ c) $h(x) = (x-4)^2$

11. $y = -2x^2 - 4x + 6 = -2(x+1)^2 + 8$
vertex: $(-1, 8)$
$x-$intercepts: $(-3, 0)$ and $(1, 0)$
$y-$intercept: $(0, 6)$



12. a) $V(4, -2)$ is a minimum
b) $V(-1, -12)$ is a maximum
c) $V(2, 5)$ is a minimum

13. a) 1024 ft
b) 1088 ft

14. a) -1 b) 20 c) -40 d) -2 e) -12 f) $10\sqrt{2} - 3$ g) 22 h) 15 i) -12
j) $0, 10$ k) $5 + 2\sqrt{6}$, $5 - 2\sqrt{6}$

15. a) $\frac{587}{900}$ b) $\frac{871}{990}$ c) $\frac{991\,917}{999\,900}$

16. 21 359

17. $\sqrt{38}$ ft

18. a) $\frac{3}{4}(x+12) = y - 7$ b) $-4(x-3) = y + 1$ or $-4(x+5) = y - 7$ or $y = -4m - 13$
c) $\frac{3}{2}(x-6) = y - 2$ or $y = \frac{3}{2}x - 7$ d) $x = -3$ e) $-\frac{2}{3}(x-6) = y - 2$ or $y = -\frac{2}{3}x + 6$

19. a) $-57, -31$ and $31, 57$ b) 10 gallons c) 42 gallons of 7% and 18 gallons of 27% d) 15 hours
e) 13% decrease f) 1575 000 g) jogger: $8\frac{\text{mi}}{\text{h}}$ bicycle: $17\frac{\text{mi}}{\text{h}}$ h) 33 hours i) 4 j) 8 seconds
k) 4 hours l) \$2000 at 6% and \$6000 at 9% m) 28 n) 95 o) 10 cm and 24 cm p) 37
q) $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$ r) 20 unit s) 48 t*) cats to dogs is 4 to 3

Last revised: November 24, 2016