

1. Expand each of the following.

a) $(5a - 2b)(2c + d)$

c) $(5\sqrt{3} - 2)(2\sqrt{3} + 1)$

e) $(x - y)(x^3 + x^2y + xy^2 + y^3)$

b) $(5x - 2)(2x + 1)$

d) $(x - y)(x^2 + xy + y^2)$

2. Based on the previous problem, can you guess how $x^5 - y^5$ can be factored?

3. Completely factor each of the following over the real numbers.

a) $3x^2 - 14x + 15$

c) $24a^5 - 6a^3$

d*) $(2a^2 - a + 1)^2 - (a^2 + a - 1)^2$

b) $ax^2 - 18m - 9a + 2mx^2$

4. Simplify each of the following.

a) $\frac{ax + ay - 2bx - 2by}{a - 2b}$

b) $\frac{4x^2 + 9x + 2}{x^2 + 2x - 8} - \frac{x - 1}{x + 4}$

c) $\frac{3x^2 - 3x}{x^2 - 25} \div \frac{x^2 + x - 2}{2x + 10}$

d) $\frac{11x - 1}{x^2 + 2x - 15} + \frac{x - 2}{x + 5}$

5. Simplify each of the following.

a) $16^{3/4}$

d) $-16^{-3/4}$

g) $-(-8)^{-1/3}$

i) $\frac{x^{1/2}x^{2/3}}{(x^{-1/12})^{-2}}$

b) $16^{-3/4}$

e) $(-16)^{3/4}$

h) $(x^{2/3})^6$

j) $(\sqrt[4]{x})^{10}$

c) $-16^{3/4}$

f) $8^{1/3}$

6. Simplify each of the following.

a) $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{b} - \frac{1}{a}}$

b) $\left(\frac{3a^2b^{-3}a^0}{-2a^{-3}b^2}\right)^{-2}$

d) $\frac{a^2 - 5a - 6}{a^2 - 10a + 16} - \frac{3}{a - 8}$

f) $(\sqrt{8} - 3)^3 (\sqrt{8} + 3)^3$

c) $3\sqrt{4}(4\sqrt{6} + 2\sqrt{2})$

e) $\frac{\sqrt{5}}{\sqrt{5} - 2}$

7. Solve each of the following equations. Make sure to check your solutions.

a) $3x^2 - 7x = 0$

f) $2 - \sqrt{3x + 1} = -1$

j) $\frac{x}{x - 4} - \frac{3}{x + 1} = \frac{4x + 4}{x^2 - 3x - 4}$

b) $3x^2 - 7x = 1$

g) $x + 4 = -2\sqrt{x + 28}$

k) $\frac{x}{x + 5} - \frac{8}{x + 3} = \frac{-8x - 12}{x^2 + 8x + 15}$

c) $6x^2 + x^3 = 567x$

h) $\sqrt{3x + 12} - \sqrt{x + 2} = 2$

l) $12x^5 = 3x^3$

d) $|3x + 1| - 3 = 5$

i) $\frac{4}{x + 3} - \frac{1}{5} = \frac{8}{4x + 12}$

m) $\frac{x}{2} - \frac{x + 1}{5} = \frac{x}{3} + 1$

e) $|x + 5| = \frac{1}{3}x - 2$

n) $(2x - 1)(x - 2) + 7 = 4 - (3 - x)(x - 5)$

8. a) Solve the equation $9x^2 - 12x = 1$.

b) Check your solution using exact values.

9. Solve each of the following system of linear equations.

a) $\begin{cases} 3x - 2y = -5 \\ 3x + y = 7 \end{cases}$

b) $\begin{cases} (x + 3)^2 - (x - 3)^2 = (y + 5)^2 - (y - 1)^2 \\ x = y - 7 \end{cases}$

c) $\begin{cases} 2x + 5y = -10 \\ -x = \frac{5}{2}y + 5 \end{cases}$

10. Solve each of the following the inequalities.

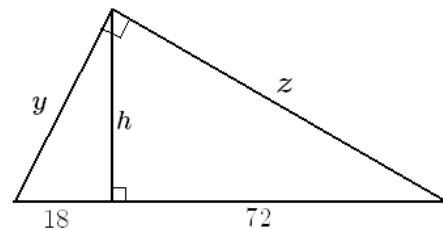
a) $\frac{3 - 4x}{3} - \frac{2x - 3}{7} \geq -x + 7$

c) $\frac{1}{4}(x - 3)^2 \leq \left(\frac{1}{2}x + 1\right)^2$

d) $-\frac{2}{3}(x + 1) > \frac{1}{2}(x - 5)$

b) $-4(x - 3) + 13 < 9$

11. Solve each of the following compound inequalities.
- a) $\frac{13-x}{5} < 4$ or $\frac{1}{3}x - 2 \leq -4$ b) $\frac{13-x}{5} < 4$ and $\frac{1}{3}x - 2 \leq -4$
12. Compute the sum $13 + 18 + 23 + \dots + 558$
13. Prove that the decimal $0.3826262626\dots$ represents a rational number by converting it to a fraction of two integers. You do not have to reduce the fraction.
14. Suppose that $f(x)$ is a function given by $f(x) = -x^2 + 8x$. Find the value of
- a) $f(0)$ c) $f(-2)$ e) $f(2) + f(3)$ g) $f(3 \cdot 1)$ i) $f(f(10))$
 b) $f(2)$ d) $f(2+3)$ f) $3f(1)$ h) $f(f(1))$
- j) Find all values of x with $f(x) = 7$ k) Find all values of x with $f(x) = 6$
15. If $f(x) = \frac{1}{2}x + 4$, find the coordinates of the
- a) x -intercept. b) y -intercept.
16. a) Find an equation of the straight line that is parallel to $y = 3x - 7$ and passes through the point $(-6, 1)$.
 b) Find an equation of the straight line that is perpendicular to $2x - 7y = 42$ and passes through the point $(2, -4)$.
 c) Find an equation of the straight line that passes through the points $(3, -8)$ and $(-1, 0)$.
17. a) Compute the perimeter and area of the triangle determined by the points $A(-4, 3)$, $B(8, 3)$, and $C(8, 12)$.
 b*) Compute the perimeter and area of the triangle determined by the points $A(-4, 3)$, $B(8, 3)$, and $C(0, 7)$.
18. Compute the exact value of the area of the triangle with sides 12 cm, 7 cm, and 7 cm long.
19. Compute the exact value of the distance between the points $A(6, -1)$ and $B(46, -10)$.
20. Find the greatest value of the expression $-5x^2 + 20x - 35$.
21. Graph the parabola $y = -x^2 + 6x$. Clearly indicate the coordinates of five points, including vertex and intercepts.
22. Compute the exact values of x , y , and z based on the given picture.



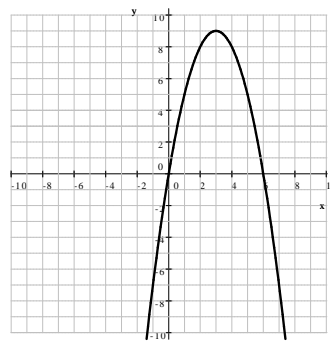
23. We traveled for 5 hours. Then we increased our velocity by 9 miles per hour and traveled an additional 4 hours. What was our original velocity if all together we have traveled 216 miles?
24. There is a farm where chickens and cows live. There are 53 heads and 160 legs. How many chickens, how many cows?
25. Compute the exact value of the height of the square-based straight pyramid, given that the base is a square with sides 14 centimeters long, and all other edges are 20 centimeters long.
26. How many milliliters of a 26% acid solution do we need to mix with 3 milliliters of an 18% acid solution so that the resulting mixture is a 23% acid solution?

27. The shortest side of a right triangle is 12 meters long. The difference between the lengths of the other two sides is 2 meters. Find the missing sides. Use exact values.
28. The population of a town is currently 60 000. What would be the population after a year if during the next year, there will be a
- a) 15% decrease in the population? b) 15% increase in the population?
29. An arch is in the shape of a semicircle. At a point along the base 6 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch. Present exact value of the answer.
30. We invested \$3600 in two bank accounts. One account earns 5% interest per year, the other earns 3% interest per year. How much did we invest into each account if the combined interest of the two accounts was \$146 after one year?
31. The sum of two numbers is 27. Their difference is 11. Find these numbers.
32. Seth and Ted can paint a room in 2.1 hours if they work together. If Ted were to work by himself, it would take him 3 hours to paint the room. How long would it take Seth to paint the room by himself if Ted calls in sick?
33. How many gallons of each of a 55% acid solution and a 30% acid solution should be mixed to obtain 75 gallons of solution that is 39%?
34. Six times a number is sixteen less than the square of a number. Find this number.
35. The digits in a two-digit number add up to 8. If we interchange the digits in the number, we obtain a new number that is 36 greater than the original number. Find the original number.
36. The dog was chasing the cat who had a 85 feet head start. The velocity of the dog was 12 feet per second while that of the cat was 7 feet per second. How long until the dog catches up with the cat?
37. One side of a rectangle is one meter shorter than four times another side. Find the sides if the area of the rectangle is 60 m^2 .
38. A bicycle leaves New York, heading West with a speed of $5 \frac{\text{mi}}{\text{h}}$. Exactly 14 hours later, a second bicycle leaves New York, heading West with a speed of $12 \frac{\text{mi}}{\text{h}}$. How long will it take for the second bicycle to overtake the first bicycle?
39. A number is $\frac{21}{10}$ greater than its own reciprocal. Find this number.
40. Kyle can paint a room in 15 hours. Jeremy can paint the same room in 9 hours. How long does it take for both Kyle and Jeremy to paint the room if they are working together?
41. The attendance in the camp was at first 300. Then, when half of the boys left the camp, the attendance dropped to 220. How many girls were at the camp?
42. A 5.4 ft tall person is standing 20 ft away from a street light that is 15 ft tall. How long is his shadow?

Answers

1. a) $10ac + 5ad - 4bc - 2bd$ b) $10x^2 + x - 2$ c) $28 + \sqrt{3}$ d) $x^3 - y^3$ e) $x^4 - y^4$
2. $x^5 - y^5 = (x - y)(x^4 + y^4 + xy^3 + x^3y + x^2y^2)$
3. a) $(3x - 5)(x - 3)$ b) $(x - 3)(x + 3)(a + 2m)$ c) $6a^3(2a - 1)(2a + 1)$ d*) $3a^2(a^2 - 2a + 2)$
4. a) $x + y$ b) $\frac{3x}{x - 2}$ c) $\frac{6x}{x^2 - 3x - 10} = \frac{6x}{(x - 5)(x + 2)}$ d) $\frac{x + 1}{x - 3}$
5. a) 8 b) $\frac{1}{8}$ c) -8 d) $-\frac{1}{8}$ e) undefined f) 2 g) $\frac{1}{2}$ h) x^4 i) x j) $\sqrt{x^5}$
6. a) $\frac{a + b}{a - b}$ b) $\frac{4b^{10}}{9a^{10}}$ c) $12\sqrt{2} + 24\sqrt{6}$ d) $\frac{a}{a - 2}$ e) $2\sqrt{5} + 5$ f) -1
7. a) $0, \frac{7}{3}$ b) $\frac{7 + \sqrt{61}}{6}, \frac{7 - \sqrt{61}}{6}$ c) 21, 0, -27 d) $-3, \frac{7}{3}$ e) no solution ($-\frac{21}{2}$ and $-\frac{9}{4}$ don't work)
- f) $\frac{8}{3}$ g) -12 (8 does not work) h) -1 i) 7 j) 2 (4 does not work) k) -7, 4 l) $-\frac{1}{2}, 0, \frac{1}{2}$
- m) -36 n) -5, 2
8. a) $\frac{2 \pm \sqrt{5}}{3}$
- b) If $x = \frac{2 + \sqrt{5}}{3}$, then
- $$\begin{aligned} \text{LHS} &= 9 \left(\frac{2 + \sqrt{5}}{3} \right)^2 - 12 \left(\frac{2 + \sqrt{5}}{3} \right) = 9 \frac{(2 + \sqrt{5})^2}{3^2} - 4(2 + \sqrt{5}) = (2 + \sqrt{5})^2 - 4(2 + \sqrt{5}) \\ &= 4 + 5 + 4\sqrt{5} - 8 - 4\sqrt{5} = 1 = \text{RHS} \end{aligned}$$
- and if $x = \frac{2 - \sqrt{5}}{3}$, then
- $$\begin{aligned} \text{LHS} &= 9 \left(\frac{2 - \sqrt{5}}{3} \right)^2 - 12 \left(\frac{2 - \sqrt{5}}{3} \right) = 9 \frac{(2 - \sqrt{5})^2}{3^2} - 4(2 - \sqrt{5}) = (2 - \sqrt{5})^2 - 4(2 - \sqrt{5}) \\ &= 4 + 5 - 4\sqrt{5} - 8 + 4\sqrt{5} = 1 = \text{RHS} \end{aligned}$$
9. a) (1, 4) b) no solution c) infinitely many solutions in the form $\left(x, -\frac{2}{5}x - 2\right)$
10. a) $(-\infty, -9]$ b) $(4, \infty)$ c) $\left[\frac{1}{2}, \infty\right)$ d) $\left(-\infty, \frac{11}{7}\right)$
11. a) \mathbb{R} b) $(-7, -6]$ 12. 31 405 13. $\frac{3788}{9900}$
14. a) 0 b) 12 c) -20 d) 15 e) 27 f) 21 g) 15 h) 7 i) -560 j) 1, 7 k) $4 - \sqrt{10}, 4 + \sqrt{10}$
15. a) $(-8, 0)$ b) $(0, 4)$ 16. a) $y - 1 = 3(x + 6)$ b) $y + 4 = -\frac{7}{2}(x - 2)$ c) $y = -2x - 2$
17. a) $P = 36$ unit and $A = 54$ unit² b*) $P = 12 + \sqrt{32} + \sqrt{80} = 4\sqrt{2} + 4\sqrt{5} + 12$ unit and $A = 24$ unit²
18. $6\sqrt{13}$ cm² 19. 41 20. -15

21. y -intercept: $(0, 0)$
 x -intercepts: $(0, 0)$ and $(6, 0)$
 vertex: $(3, 9)$
 additional points: $(1, 5)$, $(2, 8)$, $(4, 8)$, $(5, 5)$



22. $h = 36$ $y = 18\sqrt{5}$ $z = 36\sqrt{5}$ 23. $20\frac{\text{mi}}{\text{h}}$ 24. 26 chickens and 27 cows 25. $\sqrt{302}$
26. 5 milliliters 27. 35 m and 37 m 28. a) 51 000 b) 69 000 29. $\frac{25}{3}$ ft
30. 1700 at 3% and 1900 at 5% 31. 8 and 19 32. 7 hours
33. 27 gallons of the 55% acid solution and 48 gallons of the 30% acid solution
34. -2 and 8 35. 26 36. 17 seconds 37. 4 m by 15 m 38. 10 hours 39. $-\frac{2}{5}$ and $\frac{5}{2}$
40. $\frac{45}{8}$ hours = 5.625 hours 41. 140 42. $\frac{45}{4}$ ft = 11.25 ft

Last revised: November 30, 2016