SOLVING WORK-RATE PROBLEMS

Part I: Introduction

To solve work-rate problems it is helpful to use a variant of *distance equals rate times time*. Specifically:

$$Q = rt$$

In this formula Q is the quantity or amount of work done, r is the rate of work and t is the time worked.

EX 1: If a machine can produce $2\frac{1}{2}$ parts per minute then in:

4 minutes, it can produce $Q = \frac{5}{2} \cdot \frac{4}{1} = 10$ parts.

40 minutes, it can produce $Q = \frac{5}{2} \cdot \frac{40}{1} = 100$ parts.

$\frac{2}{5}$ minutes, it can produce $Q = \frac{5}{2} \cdot \frac{2}{5} = 1$ part.

(In all the calculations above the rate $2\frac{1}{2}$ has been rewritten as the improper fraction $\frac{5}{2}$.)

It is no coincidence that the machine produces 1 part in $\frac{2}{5}$ minute and that $\frac{2}{5}$ is the reciprocal of the rate $\frac{5}{2}$. To further support this, consider the next example.

EX 2: If Paul can inventory a small stockroom in 2 hours, then his rate of work is $\frac{1}{2}$ stockroom per hour. Thus in:

2 hours, he inventories $Q = \frac{1}{2} \cdot \frac{2}{1} = 1$ stockroom.

10 hours, he could inventory $Q = \frac{1}{2} \cdot \frac{10}{1} = 5$ stockrooms. (Maybe he works for a chain.)

x hours, he could inventory $Q = \frac{1}{2}x$ stockrooms.

Rate is the reciprocal of the time it takes to complete one task, whether it is building a part or inventorying a small store.

Part II: <u>Solving Problems</u>

In most cases you will be asked to find out how long it will take to perform a task if more than one worker does a portion of the task. To make the problems easier to model you will assume the workers do not get in each other's way or find a more efficient way to work as partners. Each will do his or her own part.

EX 1: Paul can complete an inventory in 2 hours. Alia can complete the same inventory in 3 hours. If they work together, how long will it take to complete one inventory?

Rate	Time	Quantity
Paul $\frac{1}{2}$	x	$\frac{1}{2}x$
Alia $\frac{1}{3}$	x	$\frac{1}{3}x$

Paul's Portion of the Inventory + Alia's Portion of the Inventory = 1 Inventory

$$\frac{1}{2}x+\frac{1}{3}x=1$$

$$6\left(\frac{1}{2}x+\frac{1}{3}x\right)=6(1)$$

$$3x+2x=6$$

$$5x=6$$

$$\frac{5x}{5}=\frac{6}{5}$$

$$x=1.2 \text{ hours}$$

Paul and Alia could complete the inventory in 1.2 hours working together.

EX 2: One printing press can produce a newspaper in 6 hours running alone. A second press could produce the paper in 9 hours running alone. A third press could produce the paper in 12 hours working alone. A late-breaking headline caused the newspaper to change the paper later than usual. If all three presses are run, how long will it take to produce the newspaper?

Rate	Time	Quantity
Fast Press $\frac{1}{6}$	x	$\frac{1}{6}x$
Medium-Speed Press $\frac{1}{9}$	x	$\frac{1}{9}x$
Slow Press $\frac{1}{12}$	x	$\frac{1}{12}x$

Fast Press's Portion + Medium-Speed Press's Portion + Slow Press's Portion = 1 Printing

$$\frac{1}{6}x+\frac{1}{9}x+\frac{1}{12}x=1$$
$$36\left(\frac{1}{6}x+\frac{1}{9}x+\frac{1}{12}x\right)=36(1)$$
$$6x+4x+3x=36$$
$$13x=36$$
$$x=\frac{36}{13}$$
$$x\approx 2.8 \text{ hours}$$

The newspaper can print enough papers in around 2.8 hours if they use all three presses.

Sometimes one of the rates is essentially negative because one person or agent undoes the work of another.

EX 3: Leto's department can deliver enough parts to fill a storeroom in 8 hours. Jessica's department uses the part to build another product. Her department can deplete a full storeroom in 12 hours. Starting with an empty storeroom, how long will it take Leto's area to fill the storeroom while Jessica's department is working?

Rate	Time	Quantity
Leto's Department $\frac{1}{8}$	x	$\frac{1}{8}x$
Jessica's Department $-\frac{1}{12}$ (Her department EMPTIES the room)	x	$-\frac{1}{12}x$

$$\frac{1}{8}x-\frac{1}{12}x=1$$
$$24\left(\frac{1}{8}x-\frac{1}{12}x\right)=24(1)$$
$$3x-2x=24$$
$$x=24 \text{ hours}$$

It would take 24 hours to fill the storeroom with parts.

EX 4: How would Example 3 change if the storeroom were half-full when both departments started working?

You might expect it would take half as long to fill- 12 hours. This is correct. To confirm it replace 1 with $\frac{1}{2}$ in the equation above since only $\frac{1}{2}$ of the storeroom remains to be filled.

$$\frac{1}{8}x-\frac{1}{12}x=\frac{1}{2}$$
$$24\left(\frac{1}{8}x-\frac{1}{12}x\right)=24\left(\frac{1}{2}\right)$$
$$3x-2x=12$$
$$x=12 \text{ hours}$$

Exercises:

1. Ghanima can correct a stack of papers in 5 hours, and Thufir can correct them in 4 hours. Howe long does it take them to correct the papers together?

2. Duncan can clean the house in 2 hours and Piter can clean the house in 3 hours. How long will it take them to clean the house together?

3. Atreides Painting Company can paint a house in 4 hours. Harkonnen Painting can paint the house in 3 hours. How long will it take them to paint the house together?

4. A cold-water pipe can fill a swimming pool in 10 hours, and a hot-water pipe can fill the pool in 15 hours. How long will it take to fill the pool if both pipes are left open?

5. Gurney can mow the lawn in $3\frac{1}{2}$ hours. Moneo can mow the lawn in $4\frac{1}{4}$ hours. How long will it take them to mow the lawn if they work together?

6. Siona can wash and wax a car in $1\frac{1}{2}$ hours. Vladimir can do the same job in 2 hours. How long will it take them to do the job together?

7. An inlet pipe can fill a barrel of vinegar in 8 hours and an outlet pipe can empty it in 12 hours. How long will it take to fill the barrel if both pipes are left open?

8. A tank can be filled by an inlet pipe in 12 hours and emptied by a drain pipe in 16 hours. How long will it take to fill the tank if both pipes are left open?

Extensions:

9. Mr. and Mrs. Corrino can complete a job in 5 hours. If Mr. Corrino works twice as long as Mrs. Corrino if each does the job alone, how long does it take Mrs. Corrino to complete the job alone?

10. Tharthar and Shoab can complete a job in 3 hours working together. If Shoab works three times as long as Tharthat if each does the job alone, how long would it take Shoab to complete the job alone?

11. One person can clean the house in 5 hours. If a second person helps to clean the house the job can be done in 2 hours. How long should it take the second person to clean the house?

12. Stilgar can mow the lawn in 3 hours. If Stilgar and Jamis work together the lawn can be mowed in 2 hours. How long would it take Jamis to mow the lawn alone?

Solutions to the odd-numbered problems and answers to the even-numbered problems:

1. $2\frac{2}{9}$ hours

$$\frac{1}{5}x+\frac{1}{4}x=1$$
$$20\left(\frac{1}{5}x+\frac{1}{4}x\right)=20(1)$$
$$4x+5x=20$$
$$9x=20$$
$$x=\frac{20}{9}$$

2. $1\frac{1}{5}$ hours

3. $1\frac{5}{7}$ hours

$$\frac{1}{4}x+\frac{1}{3}x=1$$
$$12\left(\frac{1}{4}x+\frac{1}{3}x\right)=12(1)$$
$$3x+4x=12$$
$$7x=12$$
$$x=\frac{12}{7}$$

4. 6 hours

5. $1\frac{57}{62}$ hours

The reciprocal of $3\frac{1}{2}=\frac{7}{2}$ is $\frac{2}{7}$.

The reciprocal of $4\frac{1}{4}=\frac{17}{4}$ is $\frac{4}{17}$.

$$\frac{2}{7}x+\frac{4}{17}x=1$$
$$119\left(\frac{2}{7}x+\frac{4}{17}x\right)=119(1)$$
$$34x+28x=119$$
$$62x=119$$
$$x=\frac{119}{62}$$

6. $\frac{6}{7}$ hours

7. 24 hours

8. 48 hours

$$\frac{1}{8}x-\frac{1}{12}x=1$$
$$24\left(\frac{1}{8}x-\frac{1}{12}x\right)=24(1)$$
$$3x-2x=24$$
$$x=24$$

9. $7\frac{1}{2}$ hours

10. 12 hours

This time x represents the time it takes Mrs. Corrino. Her rate is then $\frac{1}{x}$. Her husband's rate is $\frac{1}{2x}$ because his time is $2x$.

$$\frac{1}{x}(5)+\frac{1}{2x}(5)=1$$
$$\frac{5}{x}+\frac{5}{2x}=1$$
$$2x\left(\frac{5}{x}+\frac{5}{2x}\right)=2x(1) \quad \text{(LCD is } 2x\text{.)}$$
$$10+5=2x$$
$$15=2x$$
$$\frac{15}{2}=x$$

11. $3\frac{1}{3}$ hours

12. 6 hours

This time x is the time for the second person. That person's rate is $\frac{1}{x}$.

$$\frac{1}{5}(2)+\frac{1}{x}(2)=1$$
$$\frac{2}{5}+\frac{2}{x}=1$$
$$5x\left(\frac{2}{5}+\frac{2}{x}\right)=5x(1)$$
$$2x+10=5x$$
$$10=3x$$
$$\frac{10}{3}=x$$

NOTE: You can get additional instruction and practice by going to the following websites:

http://www.purplemath.com/modules/workprob.htm Purplemath (Elizabeth Stapel) explains how to do these problems by adding the rates rather than the amount of work completed.

http://www.algebralab.org/Word/Word.aspx?file=Algebra_WorkingTogether.xml AlgebraLAB (Mainland High School) has an explanation followed by a few problems to try.