If we write the number 128, we automatically know that the digit 1 denotes 100, the digit 2 denotes 20, and the digit 8 denotes 8. What do we do if we want to talk about the digits of a number?

Suppose that a, b, and c denotes digits, i.e they are integers b and c between 0 and 9, and a between1 and 9. We cannot write abc to denote this three-digit number because that means the product of the three numbers. To avoid this confusion, we denote the three-digit number by

$$\overline{abc}$$

Now that a, b, and c denote digits, we can express the number as

$$100a + 10b + c$$

just as much as 128 can be written as

$$100 \cdot 1 + 10 \cdot 2 + 8$$

Using this technique, we can solve word problems about digits in numbers.

Sample Problems

1. The sum of the digits in a two-digit number is 11. If we interchange the digits in the number, we obtain a new number that is 27 less than the original number. Find this number.
2. The one's digit in a two-digit number is three less than the ten's digit. If we interchange the digits in the number and add this new number to the original number, the sum is 77. Find this number.

Practice Problems

1. The digits in a two-digit number add up to 9. If we interchange the digits in the number, we obtain a new number that is 63 greater than the original number. Find the original number.
2. The digits in a two-digit number add up to 17. If we interchange the digits in the number, we obtain a new number that is 9 less than the original number. Find the original number.
3. The digits in a two-digit number add up to 7. If we interchange the digits in the number, we obtain a new number that is 45 less than the original number. Find the original number.
4. The one's digit in a two-digit number is 7 greater than the ten's digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 99. Find this number.
5. The ten's digit in a two-digit number is 2 greater than the one's digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 176. Find this number.
6. The ten's digit in a two-digit number is 3 less than the one's digit. If we interchange the digits in the number, we obtain a new number that, when added to the original number, results in the sum 143. Find this number.

© Hidegkuti, 2016

Answers to Sample Problems

1.) 74 2.) 52

Answers to Practice Problems

1.) 18 2.) 98 3.) 61 4.) 18 5.) 79 6.) 58

Solutions to Sample Problems

1. The sum of the digits in a two-digit number is 11. If we interchange the digits in the number, we obtain a new number that is 27 less than the original number. Find this number.

   Solution: Suppose that the two digit-number is $\overline{ab}$. This means that a and b are digits, i.e. positive integers between 1 and 9. Note that b could also be 0. Then the number $\overline{ab}$ can be expressed as $10a+b$. As we have two unknown variables, we should come up with two equations.

   The first equation will express the sum of the two digits.

$$a+b=11$$

   The second equation will express that swapping the digits will result in a number that is 27 less than the original number.

$$\overline{ab}=\overline{ba}+27$$

   We can express $\overline{ab}$ as $10a+b$ and $\overline{ba}=10b+a$. So this equation will become

$$\begin{aligned} 10a+b &= 10b+a+27 && \text{subtract } a \\ 9a+b &= 10b+27 && \text{subtract } 10b \\ 9a-9b &= 27 && \text{factor out } 9 \\ 9(a-b) &= 27 && \text{divide by } 9 \\ a-b &= 3 \end{aligned}$$

   Now our system of linear equations is really simple.

$$\begin{aligned} a+b &= 11 \\ a-b &= 3 \end{aligned}$$

   If we add the two equations, then b will cancel out and we are left with

$$\begin{aligned} 2a &= 14 && \text{divide by } 2 \\ a &= 7 \end{aligned}$$

   We can use any of the equations to find b. We will use the first equation.

$$\begin{aligned} 7+b &= 11 && \text{subtract } 7 \\ b &= 4 \end{aligned}$$

   So our number, $\overline{ab}$ is 74. Let us check if this number is indeed the one described in the problem. The sum if the digits is $7+4=11$. If we swap the digits, we get 47 which is indeed less than the original number, and the difference is $74-47=27$. Thus the number is indeed 74.

2. The one's digit in a two-digit number is three less than the ten's digit. If we interchange the digits in the number and add this new number to the original number, the sum is 77. Find this number.

   Solution: Suppose that the two digit-number is $\overline{ab}$. This means that a and b are digits, i.e. positive integers between 1 and 9. Note that b could also be 0. Then the number $\overline{ab}$ can be expressed as $10a+b$. As we have two unknown variables, we should come up with two equations.

   The first equation will express the comparison between the two digits.

$$b+3=a$$

   The second equation will express that swapping the digits to obtain a new number and then adding it to the original number results in 77.

$$\overline{ab}+\overline{ba}=77$$

   We can express $\overline{ab}$ as $10a+b$ and $\overline{ba}=10b+a$. So this equation becomes

$$\begin{aligned} 10a+b+10b+a &= 77 && \text{combine like terms} \\ 11a+11b &= 77 && \text{factor out 11} \\ 11\left(a+b\right) &= 77 && \text{divide by 11} \\ a+b &= 7 \end{aligned}$$

   Now our system of linear equations is really simple.

$$\begin{aligned} a &= b+3 \\ a+b &= 7 \end{aligned}$$

   This time we will use substitution. We substitute $a=b+3$ in the second equation.

$$a+b=7 \quad \text{becomes} \quad b+3+b=8$$

$$\begin{aligned} 2b+3 &= 7 && \text{subtract 3} \\ 2b &= 4 && \text{divide by 2} \\ b &= 2 \end{aligned}$$

   Then $a=b+3=2+3=5$. So our number, $\overline{ab}$ is 52. Let us check if this number is indeed the one described in the problem. The one's digit, 2 is indeed three less than the ten's digit, 5. Also, if we add 52 and 25, we get 77. Thus the number is indeed 52.

© Hidegkuti, 2016