Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x + 3 = 4x + 9$
2. $3w - 5 = 5(w + 1)$
3. $3y - 9 = -2y + 4$
4. $4 - x = 3(x - 7)$
5. $7(j - 5) + 9 = 2(-2j + 5) + 5j$
6. $3(x - 5) - 5(x - 1) = -2x + 1$
7. $(x - 3)^2 - (2x - 5)(x + 1) = 5 - (x - 1)^2$
8. $(x + 1)^2 - (2x - 1)^2 + (3x)^2 = 6x(x - 2)$
9. $12 - (2p - 1)(p + 1) = -2(-p + 5)^2$

Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $5x - 3 = x + 9$
2. $-x + 13 = 2x + 1$
3. $-2x + 4 = 5x - 10$
4. $5x - 7 = 6x + 8$
5. $8x - 1 = 3x + 19$
6. $-7x - 1 = 3x - 21$
7. $3(x - 4) + 5(x + 8) = 2(x - 1)$
8. $3(x - 4) = 2(x + 5)$
9. $4(5x + 1) = 6x + 4$
10. $3(2x - 7) - 2(5x + 2) = -5x - 30$
11. $a - 3 = 5(a - 1) - 2$
12. $3y - 2 = -2y + 18$
13. $2(b + 1) - 5(b - 3) = 2(b - 7) + 1$
14. $3(2x - 1) - 5(2 - x) = 4(x - 1) + 5$
15. $5(x - 1) - 3(x + 1) = 3x - 8$
16. $5(x - 1) - 3(-x + 1) = -3 + 8x$
17. $-2x - (3x - 1) = 2(5 - 3x)$
18. $3(x - 4) - 4(x - 3) = 3(x - 2) + 2(3 - x)$
19. $2x(3x - 1) - x(5x - 2) = (x - 1)^2$
20. $y^2 - (y - 1)^2 + (y - 2)^2 = (y - 3)(y - 5)$
21. $(3x)^2 - (x + 3)(5x - 3) = (5 - 2x)^2 - 16$
22. $(w + 4)(1 - 2w) = 3w - 2(w - 3)^2$
23. $(2x - 3)^2 - 3(x - 2)^2 = 10 - (x - 2)(7 - x)$
24. $(2 - w)^2 - (2w - 3)^2 + 7 = (w - 2)(5 - 3w)$
25. $3(a + 11) - a(8 - 3a) = 3(a - 2)^2$
26. $-5(2x - 1) - (4 - x)^2 = 3 - (x + 1)^2$
27. $5(-3 - x) - 3x(x - 2) = x - 3(x + 2)(x - 5)$
28. $2(-m - 2)^2 - (m - 2)^2 = 8m + (m + 2)^2$
29. $(3a - 5)(2 - a) - (2a - 1)(a + 3) = -5a^2 - 7$

Sample Problems - Answers

1.) -3 2.) -5 3.) $\dfrac{13}{5}$ 4.) $\dfrac{25}{4}$ 5.) 6 6.) no solution 7.) 2 8.) 0 9.) 3

Practice Problems - Answers

1.) 3 2.) 4 3.) 2 4.) -15 5.) 4 6.) 2 7.) -5 8.) 22 9.) 0 10.) -5 11.) 1

12.) 4 13.) 6 14.) 2 15.) 0 16.) contradiction, there is no solution 17.) 9 18.) 0 19.) $\dfrac{1}{2}$

20.) 2 21.) 0 22.) 1 23.) 3 24.) 4 25.) -3 26.) no solution 27.) -5

28.) all numbers are solution 29.) 0

Sample Problems - Solutions

1. $2x+3=4x+9$
   Solution:

$$\begin{aligned} 2x+3 &= 4x+9 && \text{subtract } 2x \text{ from both sides} \\ 3 &= 2x+9 && \text{subtract } 9 \text{ from both sides} \\ -6 &= 2x && \text{divide both sides by } 2 \\ -3 &= x \end{aligned}$$

   We check: if $x=-3$, then

$$\begin{aligned} \text{LHS} &= 2(-3)+3=-6+3=-3 \\ \text{RHS} &= 4(-3)+9=-12+9=-3 \end{aligned}$$

   Thus our solution, $x=-3$ is correct. (Note: LHS is short for the left-hand side and RHS is short for the right-hand side.)

2. $3w-5=5(w+1)$
   Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 3w-5 &= 5(w+1) \\ 3w-5 &= 5w+5 && \text{subtract } 3w \text{ from both sides} \\ -5 &= 2w+5 && \text{subtract } 5 \text{ from both sides} \\ -10 &= 2w && \text{divide both sides by } 2 \\ -5 &= w \end{aligned}$$

   We check. If $w=-5$, then

$$\begin{aligned} \text{LHS} &= 3(-5)-5=-15-5=-20 \\ \text{RHS} &= 5((-5)+1)=5(-4)=-20 \end{aligned}$$

   Thus our solution, $w=-5$ is correct.

3. $3y-9=-2y+4$
   Solution:

$$\begin{aligned} 3y-9 &= -2y+4 && \text{add } 2y \text{ to both sides} \\ 5y-9 &= 4 && \text{add } 9 \text{ to both sides} \\ 5y &= 13 && \text{divide both sides by } 5 \\ y &= \frac{13}{5} \end{aligned}$$

   We check. If $x=\dfrac{13}{5}$, then

$$\begin{aligned} \text{LHS} &= 3\left(\frac{13}{5}\right)-9=\frac{3}{1}\cdot\frac{13}{5}-9=\frac{39}{5}-\frac{9}{1}=\frac{39}{5}-\frac{45}{5}=\frac{-6}{5}=-\frac{6}{5} \\ \text{RHS} &= -2\left(\frac{13}{5}\right)+4=\frac{-2}{1}\cdot\frac{13}{5}+\frac{4}{1}=\frac{-26}{5}+\frac{20}{5}=\frac{-6}{5}=-\frac{6}{5} \end{aligned}$$

   Thus $x=\dfrac{13}{5}$ is the correct solution.

4. $4 - x = 3(x-7)$
Solution: We first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 4 - x &= 3(x-7) && \text{distribute } 3 \\ 4 - x &= 3x - 21 && \text{add } x \text{ to both sides} \\ 4 &= 4x - 21 && \text{add } 21 \text{ to both sides} \\ 25 &= 4x && \text{divide both sides by } 4 \\ \frac{25}{4} &= x \end{aligned}$$

We check. If $x = \dfrac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 4 - x = 4 - \frac{25}{4} = \frac{4}{1} - \frac{25}{4} = \frac{16}{4} - \frac{25}{4} = \frac{16-25}{4} = \frac{-9}{4} = -\frac{9}{4} \\ \text{RHS} &= 3(x-7) = 3\left(\frac{25}{4} - 7\right) = 3\left(\frac{25}{4} - \frac{7}{1}\right) = 3\left(\frac{25}{4} - \frac{28}{4}\right) = 3\left(\frac{25-28}{4}\right) \\ &= 3\left(\frac{-3}{4}\right) = \frac{3}{1} \cdot \frac{-3}{4} = \frac{-9}{4} = -\frac{9}{4} \end{aligned}$$

Thus our solution, $x = \dfrac{25}{4}$ is correct.

5. $7(j-5) + 9 = 2(-2j+5) + 5j$
Solution:

$$\begin{aligned} 7(j-5) + 9 &= 2(-2j+5) + 5j && \text{distribute on both sides} \\ 7j - 35 + 9 &= -4j + 10 + 5j && \text{combine like terms} \\ 7j - 26 &= j + 10 && \text{subtract } j \\ 6j - 26 &= 10 && \text{add } 26 \\ 6j &= 36 && \text{divide by } 6 \\ j &= 6 \end{aligned}$$

We check: if $j = 6$, then

$$\begin{aligned} \text{LHS} &= 7(6-5) + 9 = 7 \cdot 1 + 9 = 7 + 9 = 16 \\ \text{RHS} &= 2(-2 \cdot 6 + 5) + 5 \cdot 6 = 2(-12+5) + 30 = 2(-7) + 30 = -14 + 30 = 16 \end{aligned}$$

Thus our solution is correct.

6. $3(x-5) - 5(x-1) = -2x + 1$
Solution:

$$\begin{aligned} 3(x-5) - 5(x-1) &= -2x + 1 && \text{multiply out parentheses} \\ 3x - 15 - 5x + 5 &= -2x + 1 && \text{combine like terms} \\ -2x - 10 &= -2x + 1 && \text{add } 2x \\ -10 &= 1 \end{aligned}$$

Since x disappeared from the equation and we are left with an unconditionally false statement, there is no solution for this equation. This type of an equation is called a **contradiction**.

7. $(x-3)^2-(2x-5)(x+1)=5-(x-1)^2$
Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{array}{rcll}
(x-3)^2-(2x-5)(x+1) & = & 5-(x-1)^2 & \\
x^2-3x-3x+9-\left(2x^2+2x-5x-5\right) & = & 5-\left(x^2-x-x+1\right) & \text{combine like terms} \\
x^2-6x+9-\left(2x^2-3x-5\right) & = & 5-\left(x^2-2x+1\right) & \text{distribute} \\
x^2-6x+9-2x^2+3x+5 & = & 5-x^2+2x-1 & \text{combine like terms} \\
-x^2-3x+14 & = & -x^2+2x+4 & \text{add } x^2 \\
-3x+14 & = & 2x+4 & \text{add } 3x \\
14 & = & 5x+4 & \text{subtract } 4 \\
10 & = & 5x & \text{divide by } 5 \\
2 & = & x &
\end{array}$$

We check. If $x=2$, then

$$\begin{array}{rcl}
\text{LHS} & = & (2-3)^2-(2\cdot 2-5)(2+1)=(-1)^2-(4-5)(2+1)=(-1)^2-(-1)\cdot 3 \\
 & = & 1-(-3)=4 \\
\text{RHS} & = & 5-(2-1)^2=5-1^2=5-1=4
\end{array}$$

Thus 2 is indeed the solution.

8. $(x+1)^2-(2x-1)^2+(3x)^2=6x(x-2)$
Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{array}{rcll}
(x+1)^2-(2x-1)^2+(3x)^2 & = & 6x(x-2) & \\
x^2+x+x+1-\left(4x^2-2x-2x+1\right)+9x^2 & = & 6x^2-12x & \\
x^2+2x+1-\left(4x^2-4x+1\right)+9x^2 & = & 6x^2-12x & \text{distribute} \\
x^2+2x+1-4x^2+4x-1+9x^2 & = & 6x^2-12x & \text{combine like terms} \\
6x^2+6x & = & 6x^2-12x & \text{subtract } 6x^2 \\
6x & = & -12x & \text{add } 12x \\
18x & = & 0 & \text{divide by } 18 \\
x & = & 0 &
\end{array}$$

We check. If $x=0$, then

$$\begin{array}{rcl}
\text{LHS} & = & (0+1)^2-(2\cdot 0-1)^2+(3\cdot 0)^2=1^2-(-1)^2+(0)^2 \\
 & = & 1-1+0=0 \\
\text{RHS} & = & 6\cdot 0\cdot(0-2)=6\cdot 0\cdot(-2)=0
\end{array}$$

Thus 0 is indeed the solution.

9. $12-(2p-1)(p+1)=-2(-p+5)^2$
Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied,

we must keep the parentheses.

$$\begin{array}{rcll}
12-(2p-1)(p+1) & = & -2(-p+5)^2 & \\
12-\left(2p^2+2p-p-1\right) & = & -2\left(p^2-5p-5p+25\right) & \text{combine like terms} \\
12-\left(2p^2+p-1\right) & = & -2\left(p^2-10p+25\right) & \text{distribute} \\
12-2p^2-p+1 & = & -2p^2+20p-50 & \text{combine like terms} \\
-2p^2-p+13 & = & -2p^2+20p-50 & \text{add } 2p^2 \\
-p+13 & = & 20p-50 & \text{add } p \\
13 & = & 21p-50 & \text{add } 50 \\
63 & = & 21p & \text{divide by } 21 \\
3 & = & p &
\end{array}$$

We check. If $p=3$, then

$$\begin{array}{rcl}
\text{LHS} & = & 12-(2\cdot 3-1)(3+1)=12-(6-1)(3+1)=12-5\cdot 4=12-20=-8 \\
\text{RHS} & = & -2(-3+5)^2=-2\cdot 2^2=-2\cdot 4=-8
\end{array}$$

Thus 3 is indeed the solution.

