Math 98/99 Re-Design – Fall 2018

Contents

Purpose of Re-Design

Until recently, Math 98 (Beginning Algebra with Geometry) and Math 99 (Intermediate Algebra with Geometry) have been courses that were taken by both STEM (science, technology, engineering, and mathematics) and non-STEM students. The course content reflected the clients. While Math 98/99 were viewed as difficult classes, there was a noticeable gap between the content and quality of Math 99 and the next class, Math 140 (College Algebra). In short, while these courses served both STEM and non-STEM students, the courses failed to appropriately prepare STEM students for their college level classes.

With the introduction of a non-STEM developmental course, Math 90 (Mathematical Literacy) and co-requisite classes added to non-STEM college level classes, Math 98 and Math 99 became STEM-only courses. At this time, students who are taking Math 98 or Math 99 are expected to pursue a major related to mathematics.

For this reason, we need to strengthen Math 98 and Math 99 with STEM-only clients in mind. This re-design is an attempt to do that. We hope for a fall 2018 implementation for both courses.

Changes to Content

Additions to Math 98

1) The words **and** and **or** from logic.
2) Basic Set Theory (definition of a set, union, intersection, subset)
3) Basic Number Theory
4) Indirect Proof (or Proof by Contradiction)

Rationale for Additions to Math 98

1) The words **and** and **or** from logic.

When studying inequalities, students often have trouble with ≤ (read: less than or equal to), because they have trouble with the word *or*.

2) Basic Set Theory (definition of a set, union, intersection, subset)

The concept of sets is fundamental in Mathematics, all STEM student should be exposed to it. Basic set theory used to be covered in Pre-Algebra (now Foundational Studies Mathematics). When it was removed from there, it was never replaced, and only Math 118 (General Education Mathematics), a non-STEM course addresses it.

Math 98 covers inequalities, and the results are in interval notation. Intervals are special sets, and it is dishonest to present interval notation to students without the concept of sets.

Math 98 also covers the real number system, in which the set of all natural numbers, integers, rational numbers, and real numbers are sets in which the smaller set is a subset of the greater set.

Later in Math 99, we compound inequalities with solution sets of the form of $(-\infty,3)\cup(8,\infty)$. We should not present this content without the concept of sets and union.

Later in Math 140 (College Algebra), when studying functions, the domain of $f+g$ is the intersection of the domains of f and g. Again, this concept can not be presented without the concept of intersection.

3) Basic Number Theory

Technically speaking, this is not new material, just a review from FS Math (Foundational Studies Mathematics). The concept of greatest common factor are needed in factoring, the concept of least common multiple are needed in rational expressions. Prime-factorization and the fundamental theorem of arithmetic are important topics for STEM students.

The fact that √2 is irrational requires some basic number theory. The real number system is covered at the beginning of Math 98, yet students never see this proof. This is unacceptable. The proof of the irrationality of √2 is part of the course.

4) Indirect Proof (or Proof by Contradiction)

The real number system is covered at the beginning of Math 98. It is against the philosophy of this design to present facts without arguments for it. Consequently, we should not talk about rational and irrational numbers without logically convincing students about the existence of irrational numbers. Therefore, the proof of the irrationality of √2 is part of the course. This is not a difficult proof, it requires very little number theory and the concept of indirect proofs. As a similarly important and easy proof by contradiction is the proof that there are infinitely many prime numbers. This proof is also part of the course as an optional topic.

Additions to Math 99

1) All additions to Math 98
2) Basic Combinatorics (systematic listing, fundamental counting principle, and the handshake problem)
3) Summation
4) Converting a Repeating Decimal to a Quotient of Integers
5) Radical Equations more complicated than $\sqrt{(ax+b)}=c$
6) (Optional) More Set Theory (concept of cardinality and complement)

Rationale for Additions to Math 99

1) All additions to Math 98

   Most Math 99 students test into Math 99 and so will need these topics just added toMath 98. Also, both re-designs will be implemented in the fall semester of 2018. Consequently, these topics will be repetitions for none of the fall 2018 Math 99 students. In the Spring 2019 semester, a modified Math 99 will be needed to be adopt in which these topics will have to be included as review for entering students. By this time, videos and problem sets will be needed to assist students in reviewing these topics.

   The Department needs to work on further modifying Math 99 for the spring 2019 adoption.

2) Basic Combinatorics (systematic listing, fundamental counting principle, and the handshake problem)

   These topics are currently covered in Math 118, a non-STEM course. Probability, permutations, combinations, and the binomial theorem are supposed to be covered in Math 140 (College Algebra), but this is only realistic if we start the foundations in the previous course, which is Math 99. Currently, non-STEM students have a better exposure to these topics than our STEM students which is wildly inappropriate.

   These topics are playful, hands-on, concrete, and enjoyable for students. Furthermore, they provide many opportunities for connections. For example, given sets D{1,2,3,4} and R{A,B}, how many functions are possible with domain D and range R? Another example is the number of divisors of an integer, given its prime factorization.

3) Summation

   Summation will refer to computing sums such as 5+12+19+…+208 without any mentioning of arithmetic sequences or its sum formula. This will prepare students for arithmetic sequences in the next course, Math 140 (College Algebra). This is a hands-on, concrete problem that is enjoyable students. This is also a topic with many real-world applications.

4) Converting a Repeating Decimal to a Quotient of Integers

This topic belongs to FS Math (Foundational Mathematics) but it is not covered there or students forget it by the time they get to Math 99. The idea that repeating decimals can be converted into fractions of integers is fundamental to see the decimal presentation of irrational numbers. This conversation is essential when discussing approximations and exact values, a fundamental concept in algebra. There is a logical need for this before starting radical expressions.

5) Radical Equations more complicated than $\sqrt{(ax+b)}=c$

Solving such equations is necessary for success in calculus and are not covered in Math 140 (College Algebra). They also provide non-trivial examples for extraneous solutions - a result of non-equivalent steps.

6) (Optional)More Set Theory (concept of cardinality and complement)

These concepts are extremely useful in mathematics. Cardinality is an easy concept (as long as we do not bring in infinite sets) and gives us lots of opportunities for connections.

Changes to Pedagogy

Fundamental Themes

Many attributes of this re-design are inspired by the learning theory called ***constructivism***. The main idea of constructivism is to re-construct the content in order to present it in incremental steps that maximally support understanding. If done well, in a constructivist class, students often derive methods and formulas themselves. The philosophy behind constructivism is the idea that knowledge is not a static thing to place into students' mind. Instead, knowledge is constructed in the minds of students. This is particularly true for mathematics. If students themselves derive material, recall becomes much easier as we can just derive a formula on the spot when needed. Also, it enables students to actively engage with the material as opposed to passively absorbing it.

The most important theme is ***reasoning***. Student learning outcomes (SLOs) are presented in such a way that new material is easily derived from already mastered material. Once students understand the material, it is much easier to recall methods and formulas.

Every statement will ***proved*** or at least argued for it. This is fundamental to mathematics. For example, we will not formally prove that repeating decimals represent rational numbers. But students will master the technique. As a consequence, they will be 'mathematically convinced' that the statement is generally true. Further, this leads to the proof for the decimal presentation of irrational numbers. We also will reintroduce and reinforce enough number theory to prove the irrationality of $\sqrt{2}$. In the rare cases when such an argument cannot be presented, (for example, the area formula for the circle) students will be told as such, and an outline of the higher mathematical approach will be presented.

Another, related theme will be an ***emphasis on concepts and reasoning*** and a ***discouragement of memorization without understanding***. For example, the distance formula is complicated enough so that students who only memorize it recall it incorrectly. This course will instead approach this formula as an application of the Pythagorean theorem. Cute poems or memo-techniques void of mathematical reasoning will be avoided. We will aim that students internalize the reasons behind a formula – that makes recall much more efficient and precise.

Similarly, we will avoid visual or overly general clues. For example, '*two minuses makes a plus*' is not something we will use, because it is often false. Instead, we will state: "*To subtract is to add the opposite*". This kind of conceptual clues will enable students to apply these concepts correctly. In short, ***all presentations will be concept-oriented***.

The ultimate goal is the mastery of problem solving. This skill requires tenacity. Students will aggressively attack mathematics problems only if they have the confidence that they can solve the problem. In order to support and encourage tenacity, students will be presented with problems that can be solved within the scope of already presented material. ***Problems that require content beyond the scope of mastered material must not be presented to students as regular assignment.*** Similarly, instructors and tutors will be strongly urged to present content only within the scope of mastered material. We can of course tell students about interesting open questions (for example, the Goldbach conjecture) but never as a regular assignment. Presenting a carefully designed artificial 'textbook world' to students in which all problems are solvable by them will help increasing their confidence in their problem solving skills.

Reinforcement of Skills from Previous Courses

It is subject to debate whether the final exam for Math 99 should be a subject test evaluating mastery of topics within the course, or an overall algebra test, assessing whether the student is ready for college-level STEM courses and science courses. We have aimed to design final exams that were cumulative in nature, i.e. they included SLOs from Foundational Mathematics and Math 98 courses. This re-design will constantly reinforce all pre-requisite skills that were covered in FS Mathematics and Math 98. These topics include number theory, percents, basic geometry, and other, fundamental skills that are necessary for a rigorous mathematical foundation. Historically, we avoided reinforcing these topics because it often necessitated re-teaching them and we did not have the time. However, avoidance of pre-requisite skills is guarantee that students will not retain these fundamental skills. As a result of this new practice, it will be extremely important to provide students with supplemental material that enables students to review these topics outside of the regular classes. These material should include written explanations, video explanations, and practice problems.

Cumulative Assignments

All assignments and assessments should be cumulative. Repetition, practice, or reinforcement is extremely important in mastering mathematics. However, it only works we do it the correct way. One homework assignment should not be repetitious: we will not assign the same problem to students over and over in one assignment. Instead, once a concept is introduced, it must come up in every assignment. Homework assignments should therefore look like a sample final exam based on that point in the course. This practice has many advantages, including the constant practice of deciding what method is needed to solve an individual problem.

Changes to Course Outline

If student learning outcome (SLO) B is built on student learning outcome A, then the two SLOs must be separated by at least one homework assignment. This way students will arrive to class after having SLO A reinforced, students will have a solid foundation upon which to build. This might result in a course outline that looks chaotic: topics are stretched out to many classes, and a single class will address just a single SLO from a topic, but will cover several different topics. This is clearly a change from recent practice. It is important for students to keep up with the course. Instructors need to inform students and emphasize the need for keeping up with the class.

Example:
SLO A: evaluating an algebraic expression
SLO B: checking whether a number is a solution of an inequality or equation or not
SLO C: solving a linear equation

In most classes, SLO B and C would be presented at the same time. As a result, students confuse the two. We will address this by sharply separating the two outcomes. This is not just by separation in time. When presenting checking solutions, we will use quadratic or cubic equations that can not yet be solved by students. This has the advantage of forcing students to check. Also, it prepares students for the idea that not every equation has just a single solution.

Stronger Focus on Coordinate Geometry

Math 99, Intermediate Algebra with Geometry contains the computational skills that are needed in calculus. Historically, coordinate geometry used to be covered in intermediate algebra, and quickly reviewed in college algebra (Math 140). Currently, coordinate geometry is omitted from the curriculum, and so the connections between algebra and geometry are a sudden shock to students. Math 99 will not introduce extra topics (such as the circles) but will treat the current material with an emphasis on coordinate geometry.

Combination Problems

The best way of repetition is the one that students don’t even notice. The design will aim to include many problems that combine two or more SLOs in a novel way. For example, we will review plotting points on a rectangular coordinate system several classes before we start graphing lines. From that moment on, geometry problems can be stated using coordinates: given the coordinates of the vertices of a rectangle, compute the area and perimeter of the rectangle.

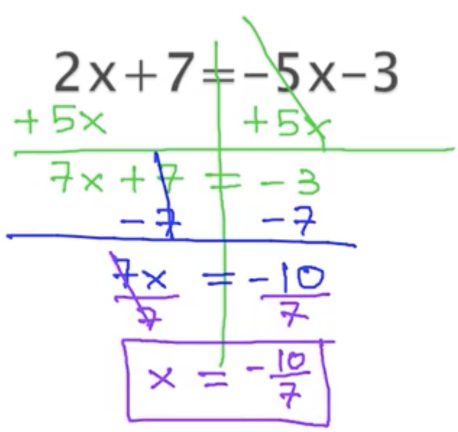
Embedded Applications

In recent practice, only a section or two covered application problems (i.e. word problems). The new design abandons this practice. Most topics will be presented with applications. This way students will constantly practice applications.

Notes on Notation

There is a subtle but important difference between teaching students how to think in a situation and how to write in the same situation. In terms of notation, instructors must keep in mind that these students will be taking calculus within two years. Instructors are asked to teach and model notation that is clear and easy to read. Computations that are busy with arrows, helper lines, visual cancellations are very difficult to read. Probably this is the reason why, in college level courses, these visual clues are omitted. When presenting material, instructors will be asked to use notation that is presented in college algebra and calculus books - and not in developmental education courses. In higher level mathematics, notation is clear.

This format is hard to read and no student in college algebra or calculus would benefit from it. Please avoid.



Please present material in this format. All higher level computations look like this in all text books.

$$\begin{aligned} 2x+7 &= -5x-3 && /+5x \\ 7x+7 &= -3 && /-7 \\ 7x &= -10 && /\div 7 \\ x &= -\frac{10}{7} \end{aligned}$$