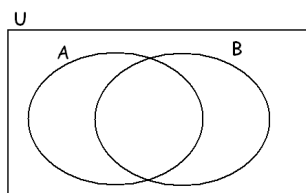1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4, 5\}$, and $B = \{1, 4, 9\}$. Find each of the following.

     a) $A \cap B$ b) $A \cup B$

2. Consider the Venn-diagram shown on the picture, depicting sets A and B in a universal set U. Shade the region corresponding to each of the following sets.

     a) $A \cap B$ b) $A \cup B$

3. Compute each of the following sets.

     a) $(1, 5) \cup [2, 8]$ e) $(3, 4) \cup [1, 7]$

     b) $(1, 5) \cap [2, 8]$ f) $(3, 4) \cap [1, 7]$

     c) $(-\infty, 5) \cup (-2, \infty)$ g) $(-\infty, -3) \cup (-1, 12)$

     d) $(-\infty, 5) \cap (-2, \infty)$ h) $(-\infty, -3) \cap (-1, 12)$

4. Perform the division with remainder.
$$2018 \div 99$$

5. a) List all factors of 54.

     b) What is the greatest factor of 54?

     c) What is the greatest prime factor of 54?

6. Find the prime factorization of 528.

7. Find the prime factorization for each of the following.

     a) 24^{100} b) $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10$

8. Use prime factorization to find the least common multiple and greatest common factor of 270 and 600.

9. Label each of the following as true or false.

a) If the integer n is divisible by 6 and the integer m is divisible by 4, then the product nm is divisible by 24.

b) If an integer n is divisible by 6 and by 4, then it is also is divisible by 24.

c) If a right triangle's sides are integers, then at least one of them must be even.

d) If n is an integer such that n^2 is divisible by 12, then n is divisible by 12.

Let T be the set of all triangles, R the set of all right triangles, and S the set of all isosceles triangles. Then

     e) $R \cup S = T$ f) $R \subseteq T$ g) $R \cap S = \varnothing$

10. Simplify each of the following. Show all steps.

     a) $\dfrac{-5^2 - 60 \div (-4) \cdot 3}{|-21 \div (-7)| - (-1)^6}$

     b) $\dfrac{\frac{1}{5} + \left(-\frac{1}{2}\right)^2 \cdot \left(3 + \frac{2}{5}\right)}{\frac{1}{5} + \frac{1}{2}} - 2^{-1}$

     c) $\dfrac{2^{-1} - 3^{-1}}{2^{-2} + 3^{-2}}$

11. Perform the given operations on the radical expressions and simplify. Use exact values.

     a) $\left(3\sqrt{2} - 1\right)\left(5\sqrt{2} + 7\right)$ f) $\sqrt{50} - 2\sqrt{8} + \sqrt{18}$

     b) $\left(\sqrt{5} - 2\right)^2$ g) $\left(\sqrt[4]{3}\right)^8$

     c) $\left(\sqrt{5} - 2\right)^3$

     d) $\left(\sqrt{3} - 1\right)^3 \left(\sqrt{3} + 1\right)^3$

     e) $\left(\sqrt{5} + 1\right)^2 - \left(\sqrt{5} - 1\right)^2$

12. Simplify each of the given expressions. Use exact values.

     a) $27^{-2/3}$ b) $(-32)^{-1/5}$ c) $\left(\dfrac{1}{9}\right)^{-3/2}$

13. The following expressions are all equal to each other except for one. Which one?

     A) $\sqrt{50} - \sqrt{8}$ C) $3\sqrt{2}$ E) $\dfrac{\sqrt{72}}{2}$

     B) $\sqrt{2} + \sqrt{12}$ D) $\sqrt{18}$

14. Rationalize the denominator in each of the given expressions.

     a) $\dfrac{3}{\sqrt{10} + 1}$ c) $\dfrac{1}{\sqrt{5} - 2}$ b) $\dfrac{6}{2 - \sqrt{7}}$ d) $\dfrac{\sqrt{8} - \sqrt{5}}{\sqrt{8} + \sqrt{5}}$

15. Prove that each of the given repeating decimals represents a rational number by re-writing it as a quotient of two integers. You do not have to reduce the fraction.

     a) $0.31\overline{7} = 0.317\,777\,777...$

     b) $0.3\overline{17} = 0.317\,171\,717...$

     c) $0.\overline{317} = 0.317\,317\,317...$

16. Compute each of the given sums.

a) $158 + 163 + 168 + ... + 398$ b) $49 + 53 + ... + 229$

17. Perform the indicated operations on the complex numbers. Simplify your answer.

a) $(2 - 3i) + (8 - i)$

b) $(2 - 3i) - (8 - i)$

c) $2i(2 - 3i)$

d) $(2 - 3i)(8 - i)$

e) $(2 - 3i)^2$

f) $(5 + 2i)(5 - 2i)$

g) $\dfrac{7 + i}{2 + i}$

h) $|5 - 2i|$

i) $(2 - i)^4$

j) i^{39}

k) $(1 - 3i)^3 (1 + 3i)^3$

18. Compute the exact value of $-p^2 + 4p - 1$ if

a) $p = -1 + 2\sqrt{3}$ b) $p = -1 + 2i$

19. Compute the exact value of $-2x^2 + 3x - 8$ if

a) $x = -\dfrac{2}{3}$ c) $x = 5 - 2i$

b) $x = -\sqrt{3} + 1$ d) $x = 1 - i\sqrt{2}$

20. Expand each of the following expressions.

a) $(5x - 2)^2$ d) $(5 - 2i)^2$

b) $(5x - 2y)^2$ e) $\left(5 - 2\sqrt{3}i\right)^2$

c) $\left(5 - 2\sqrt{3}\right)^2$

21. Simplify each of the following. Assume that all variables represent positive numbers.

a) $\left(-\dfrac{x^3 y^0 x^{-5}}{y^{-3}}\right)^{-2}$ c) $\left(\dfrac{-3p^2 q^{-5} p^{-1}}{12 pq p^{-1}}\right)^0$

b) $\dfrac{\left(-x^3 y^{-2}\right)^4 yx^{-1} \left(-2xy^{-3}x^{-2}\right)^{-1}}{x^{-3} y^0 \left(-3x^{-5}y^2\right)^{-2} y^{-3}}$

22. Completely factor $2x^2 - 50$

a) over the integers

b) over the real numbers

c) over the complex numbers

23. Completely factor $3x^2 - 21$

a) over the integers

b) over the real numbers

c) over the complex numbers

24. Completely factor $5x^2 + 20$

a) over the integers

b) over the real numbers

c) over the complex numbers

25. Completely factor $3x^2 + 6$

a) over the integers

b) over the real numbers

c) over the complex numbers

26. Completely factor $5x^4 - 80$

a) over the integers

b) over the real numbers

c) over the complex numbers

27. Completely factor each of the following over the real numbers.

a) $-3x^3 + 12x$ c) $2x^3 + 3x^2 - 2x - 3$

b) $-6x^2 - 7x + 3$

28. Simplify each of the given rational expressions.

a) $\dfrac{2x - 5}{5 - 2x}$ b) $\dfrac{x^3 - x}{x + 1}$ c) $\dfrac{x^2 + 2x - 15}{x^2 - 8x + 15}$

29. Perform the indicated operations on the given rational expressions. Simplify your answer.

a) $\dfrac{x^2 - 9}{x^2 - 3x} \div \dfrac{x^2 + 8x + 15}{2x + 10}$

b) $\dfrac{2m - 1}{m^2 - m - 2} - \dfrac{1}{m + 1}$

c) $\dfrac{x - 5}{x + 2} - \dfrac{3}{2 - x} - \dfrac{14 - x}{x^2 - 4}$

30. Simplify each of the given complex fractions.

a) $\dfrac{\dfrac{3}{x - 1} - 1}{\dfrac{2}{x - 1} + 1}$ b) $\dfrac{2a - \dfrac{1}{8a}}{4 + \dfrac{1}{a}}$ c) $\dfrac{3 + x^{-1}}{9 - x^{-2}}$

31. Simplify each of the given expressions. Assume that all variables represent positive numbers. Express your answer using only positive integer exponents.

a) $\dfrac{p^{5/2} p^{2/3}}{p^{1/6}}$ b) $\sqrt[4]{\dfrac{x^{12} y^{18} z^{24}}{x^4 y^6 z^8}}$ c) $\sqrt[3]{\dfrac{x^{11} y^5}{x^2 y^2}}$

32. Simplify each of the given radical expressions.

a) $\left(\sqrt{x} - 2\right)^2$ b) $\left(\sqrt{x - 2}\right)^2$ c) $\sqrt[6]{x^3}$ d) $\left(\sqrt[3]{x}\right)^{18}$

33. Re-write $x^{-8/3}$ using only positive integer exponents.

34. Use exponential notation to simplify each of the following. Assume that all variables represent positive numbers.

   a) $\sqrt[3]{x}\sqrt[4]{x}$ b) $\sqrt[3]{\sqrt[4]{x}}$ c) $\sqrt[5]{x\sqrt{x}}$

35. Solve each of the following the equations over the complex numbers.

   a) $3(x+2)^2 = 12 + (x-1)(4+x)$

   b) $\dfrac{3x-4}{6} - \dfrac{x+6}{4} = \dfrac{4}{3}$

   c) $2(x-1)^2 = 8 - (x+2)(3-x)$

   d) $y^5 = 9y^3$

   e) $y^5 = 9y^4$

   f) $x^2 = 6x + 1$

   g) $(x-3)^2 + 2x = 4$

   h) $\dfrac{2x+3}{5} - \dfrac{x-5}{3} = 1$

36. Solve each of the following equations over the real numbers.

   a) $3 + |4x - 6| = 9$

   b) $|2x - 5| + 2 = 11$

   c) $|x + 1| = |3x - 1|$

   d) $\sqrt{2x - 1} = 5$

   e) $\sqrt{2x - 5} = x - 4$

   f) $\dfrac{3}{p-7} + \dfrac{p+7}{p} = \dfrac{7p - 28}{p(p-7)}$

37. Solve each of the given system of linear equations.

   a) $\begin{cases} 2x - 3y = 16 \\ x + 8y = -87 \end{cases}$ c) $\begin{cases} 2x - 6y = -8 \\ x = 3y + 1 \end{cases}$

   b) $\begin{cases} 5x - 2y = 10 \\ x - 2 = \dfrac{2y}{5} \end{cases}$

38. Which set is depicted on the picture? State your answer in interval form.
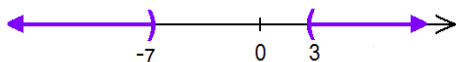
39. Solve each of the following inequalities. Present your answer using interval notation.

   a) $3(x-2) - 2(3x+4) \le -2(x+4) - 1$

   b) $(2x - 1)^2 \ge (x-2)(4x-5)$

   c) $-\dfrac{3}{5}x + 1 > -\dfrac{1}{2}x - 3$
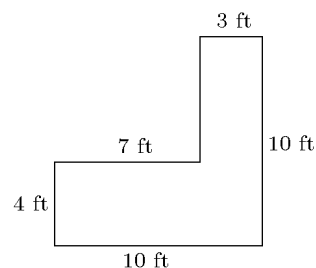
40. Solve each of the following compound inequalities.

   a) $-3 < \dfrac{1}{4}x + 2 \le 3$
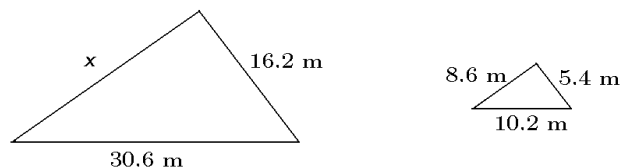
   b) $-2 \le \dfrac{1}{3}x - 1 \le 5$

   c) $3 - x \ge -2x + 5$ and $2(x-7) < x + 4$
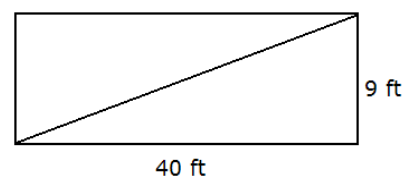
   d) $3 - x \ge -2x + 5$ or $2(x-7) < x + 4$

41. Compute the area of the object shown on the figure.

42. The two triangles shown on the picture are similar. Find the length of x.

43. Find the exact value of the length of the diagonal of the rectangle shown on the picture.

44. Find the distance between the points $P(-4, -5)$ and $Q(1, 7)$.

45. Compute the perimeter and area of the parallelogram determined by the points $A(-5, -3)$, $B(2, -3)$, $C(-2, 1)$ and $D(5, 1)$.

46. Compute the exact value of the height of the pyramid shown. Units are in meters.
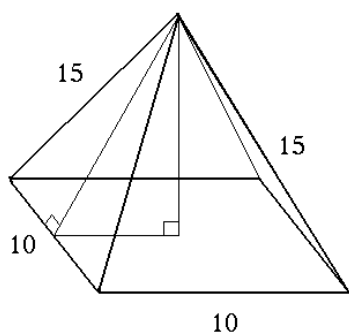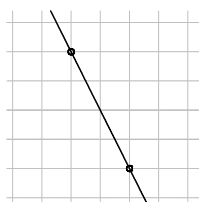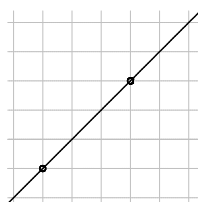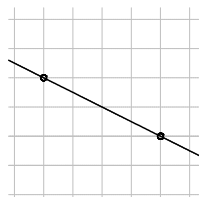

47. Find the point-slope form equation of the line that passes through the point $(-2, 3)$ and has slope $\frac{4}{5}$.

48. (Multiple choice. Select the best answer.) If a line rises from left to right, the line has a slope that is:
A) positive   C) zero   E) parallel
B) negative   D) undefined

49. Assume that the spacing is the same on both axes. Which of the lines shown below has a slope of $-\frac{1}{2}$?
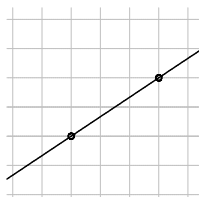
A   B
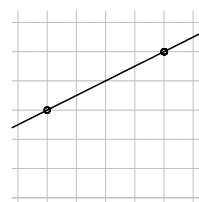
C   D   E

50. Compute the slope of the line that passes through the points $(3, 1)$ and $(6, -3)$.

51. a) Find the slope-intercept form of the equation of the straight line that is parallel to the line $y = -2x + 3$ and passes through the point $P(-6, 1)$.

    b) Find the slope-intercept form of the straight line that is perpendicular to the line $y = -2x + 3$ and passes through the point $P(-6, 1)$.

    c) Find the slope-intercept form of the straight line that passes through the points $A(-2, 3)$ and $B(4, 0)$.

52. Consider the triangle determined by $A(-4, 1)$, $B(3, 3)$, and $C(1, 9)$.

    a) Find the exact value of the length of line segment AC.

    b) Find the equation of the line that is the height belonging to side AB.

53. Find an equation for the parabola graphed.
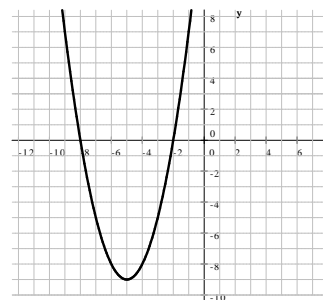
54. In each case, find both coordinates of the vertex of the parabola determined by the given equation.

   a) $y = x^2 - 6x + 5$ b) $y = 2x^2 - 20x + 18$.

55. Sketch the graph of $y = -2x^2 + 12x - 10$.

56. a) Suppose that f is a function given by $f(x) = -x^2 - x + 2$. Compute the exact value of $f(-3)$.

    b) Suppose that g is a function given by $g(x) = \sqrt{2x - 1}$. Compute the exact value of $g(13)$.

57. a) List all $x-$intercepts of the function graphed.

   b) List all $y-$intercepts of the function graphed.
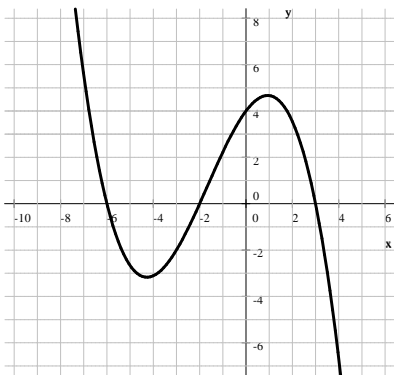
58. The picture shows the graph of a function, $f(x)$. Based on its graph, determine each of the following.

   a) the domain of f

   b) the range of f.
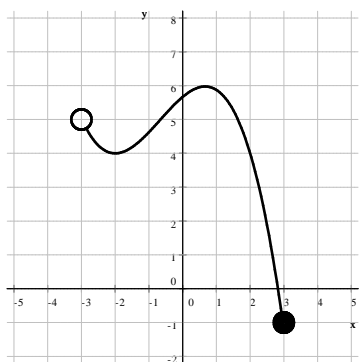
59. Suppose that our workplace pays the same amount of money for each hour worked. Based on the table shown, how much money will we make if we work for 7 hours?

Hours	payment
2	$30
4	$60
6	$90
7	

60. A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is $610. How much of each denomination of bill does he have?

61. You have a 13 inches long piece of plywood that you can use to make a ramp from the street to the top of the curb. If you only have 12 inches of horizontal space on the street, how tall is the curb?

62. After a 20% inrease, the budget was $268 800. What was the budget before the increase?

63. The hypotenuse of a right triangle is 74 ft. The difference between the other two sides is 46 ft. Find the sides of the triangle.

64. We have invested a total of $5000 in two bank accounts. One account earns 7% interest per year; the other earns 8% per year. After one year, the combined interest from the two account was $382. How much money did we invest at 8%?

65. The area of a rectangle is $1260 \, \text{m}^2$. Find the dimensions of the rectangle if we know that one side is 48 m longer than three times the other side.

66. a) A conference had 200 attendees. If every attendee shook hands with all other attendees, how many handshakes took place?

   b) 120 of the attendees were women and 80 of them were men. If men only shook hands with men and women only shook hands with women, how many handshakes took place?

   c) On the afternoon, people broke into smaller groups as there were several choices, including games and sporting events. On the men's running race, first all participants shook hands with all other participants. Later on, 5 more people joined the race. They too shook hands with all other attendees. If the number of handhsakes increased by 220 due to the new arrivals, how many men were participating in the men's running race?

67. The first row in the auditorium had 32 seats. The second row had 36 seats, the fourth row had 40 seats, and so on, each row had four more seats than the previous one. If the last row seats 100 people, then how many seats are there in the entire auditorium?

68. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens, how many cows?

69. We are standing on the top of a 896 feet tall building and launch a small object upward. The object's vertical position, measured in feet, after t seconds is

$$h(t) = -16t^2 + 160t + 896$$

    a) What is the highest point that the object reaches?
    b) How long until the object hits the ground?

70. How many liters of a 17% acid solution should be mixed with 8 liters of an 11% acid solution to obtain a mixture that is 15%?

71. A 6.4 ft tall person is standing 31 ft away from a street light that is 25 ft tall. How long is his shadow?

72. Eight times a number is nine less than the square of the number. Find this number.

73. Ann invested $20,000 in two accounts. She took a 4% loss on one of the accounts and made a 12% profit on the other investment, but ended up breaking even. How much money did she lose in the first investment?

Answers

1. a) $\{1,4\}$ b) $\{1,2,3,4,5,9\}$

2.   a)   b)
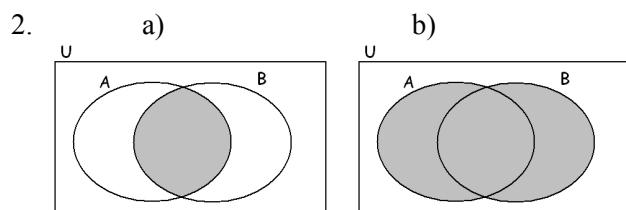
3. a) $(1,8]$ b) $[2,5)$ c) $(-\infty,\infty)$ d) $(-2,5)$
    e) $[1,7]$ f) $(3,4)$
    g) $(-\infty,-3)\cup(-1,12)$ cannot be simplified
    h) $\varnothing$ 4. 20 R 38

5. a) $1,2,3,6,9,18,27,54$ b) 54   c) 3

6. $528 = 2^4 \cdot 3 \cdot 11$

7. a) $2^{300} \cdot 3^{100}$ b) $2^8 \cdot 3 \cdot 5$

8. $\gcd(270,600) = 30$ $\operatorname{lcm}(270,600) = 5400$

9. a) true   b) false   c) true   d) false
    e) false   f) true   g) false

10. a) 10   b) 1   c) $\dfrac{6}{13}$

11. a) $23 + 16\sqrt{2}$ b) $9 - 4\sqrt{5}$ c) $-38 + 17\sqrt{5}$
    d) 8   e) $4\sqrt{5}$ f) $4\sqrt{2}$ g) 9

12. a) $\dfrac{1}{9}$ b) $-\dfrac{1}{2}$ c) 27   13. B

14. a) $\dfrac{\sqrt{10}-1}{3}$ b) $-2\sqrt{7}-4$ c) $\sqrt{5}+2$ d) $\dfrac{13-4\sqrt{10}}{3}$

15. a) $\dfrac{286}{900}$ b) $\dfrac{314}{990}$ c) $\dfrac{317}{999}$ 16. a) $13\,622$ b) 6394

17. a) $10 - 4i$ b) $-6 - 2i$ c) $6 + 4i$ d) $13 - 26i$
    e) $-5 - 12i$ f) 29   g) $3 - i$ h) $\sqrt{29}$
    i) $-7 - 24i$ j) $-i$ k) 1000

18. a) $-18 + 12\sqrt{3}$ b) $-2 + 12i$

19. a) $-\dfrac{98}{9}$ b) $-13 + \sqrt{3}$ c) $-35 + 34i$
    d) $-3 + i\sqrt{2}$

20. a) $25x^2 - 20x + 4$ b) $25x^2 - 20xy + 4y^2$
    c) $37 - 20\sqrt{3}$ d) $21 - 20i$ e) $13 - 20i\sqrt{3}$

21. a) $\dfrac{x^4}{y^6}$ b) $-\dfrac{9}{2}x^5y^3$ c) 1

22. a) $2(x+5)(x-5)$ b) $2(x+5)(x-5)$
    c) $2(x+5)(x-5)$

23. a) $3(x^2 - 7)$ b) $3(x+\sqrt{7})(x-\sqrt{7})$
    c) $3(x+\sqrt{7})(x-\sqrt{7})$

24. a) $5(x^2+4)$ b) $5(x^2+4)$ c) $5(x+2i)(x-2i)$

25. a) $3(x^2+2)$ b) $3(x^2+2)$
    c) $3(x+\sqrt{2}\,i)(x-\sqrt{2}\,i)$

26. a) $5(x^2+4)(x+2)(x-2)$
    b) $5(x^2+4)(x+2)(x-2)$
    c) $5(x+2i)(x-2i)(x+2)(x-2)$

27. a) $-3x(x-2)(x+2)$
    b) $-(2x+3)(3x-1)$
    c) $(2x+3)(x-1)(x+1)$

28. a) -1 b) x^2-x c) $\dfrac{x+5}{x-5}$

29. a) $\dfrac{2}{x}$ b) $\dfrac{1}{m-2}$ c) $\dfrac{x-1}{x+2}$

30. a) $\dfrac{-x+4}{x+1}$ b) $\dfrac{4a-1}{8}$ c) $\dfrac{x}{3x-1}$

31. a) p^3 b) $x^2y^3z^4$ c) x^3y

32. a) $x-4\sqrt{x}+4$ b) $x-2$ c) $\sqrt{x}$ e) x^6

33. $\dfrac{1}{\left(\sqrt[3]{x}\right)^8}$ 34. a) $\sqrt[12]{x^7}$ b) $\sqrt[12]{x}$ c) $\sqrt[10]{x^3}$

35. a) $-4, -\dfrac{1}{2}$ b) 14 c) $0, 3$ d) $-3, 0, 3$ e) $0, 9$
    f) $3-\sqrt{10},\ 3+\sqrt{10}$ g) $2-i,\ 2+i$ h) -19

36. a) $0, 3$ b) $-2, 7$ c) $0, 1$ d) 13 e) 7 f) -3

37. a) $(-7, -10)$
    b) There are infinitely many solutions in the form
       of $\left(x, \dfrac{5}{2}x - 5\right)$. This is a dependent system.
    c) No solution; this system is inconsistent.

38. $(-\infty, -7) \cup (3, \infty)$

39. a) $[-5, \infty)$ b) $[1, \infty)$ c) $(-\infty, 40)$

40. a) $(-20, 4]$ b) $[-3, 18]$ c) $[2, 18)$ d) $(-\infty, \infty)$

41. $58\,\text{ft}^2$ 42. $25.8\,\text{m}$ 43. $41\,\text{ft}$ 44. 13 units

45. $P = 24$ unit $A = 28\ \text{unit}^2$ 46. $\sqrt{175}\,\text{m} = 5\sqrt{7}\,\text{m}$

47. $y - 3 = \dfrac{4}{5}(x+2)$ 48. A   49. B   50. $-\dfrac{4}{3}$

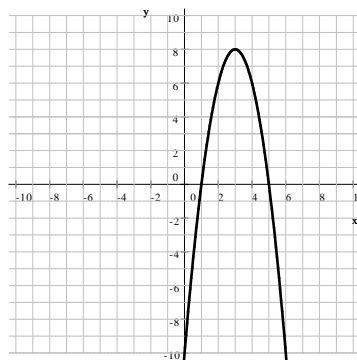51. a) $y = -2x - 11$ b) $y = \dfrac{1}{2}x + 4$
    c) $y = -\dfrac{1}{2}x + 2$

52. a) $\sqrt{89}$ unit   b) $y = -\dfrac{7}{2}x + \dfrac{25}{2}$

53. a) $(-3, 3]$ b) $[-1, 6]$ 54. $y = x^2 + 10x + 16$

55. a) $(3, -4)$ b) $(5, -32)$

56. $y = -2x^2 + 12x - 10$

57. a) -4 b) 5 58. a) $-6, -2, 3$ b) 4 59. \$105

60. 33 tens and 56 fives   61. $5\,\text{in}$ 62. \$224\,000

63. $24\,\text{ft}$ and $70\,\text{ft}$ 64. \$3200   65. $14\,\text{m}$ by $90\,\text{m}$

66. a) $19\,900$ b) $10\,300$ c) 42 67. 1188

68. 19 chickens and 34 cows

69. a) $1296\,\text{ft}$ b) 14 seconds   70. 16 liters

71. $\dfrac{32}{3}\,\text{ft} = 10.\overline{6}\,\text{ft}$ 72. -1 and 9 73. \$600