

1. If L has equation $ax + by = c$, M is its reflection across the y -axis, and N is its reflection across the x -axis, which of the following must be true about M and N for all nonzero choices of a , b , and c ?
- A. the x -intercepts are equal B. the y -intercepts are equal C. the slopes are equal
D. the slopes are opposite E. the slopes are reciprocals
2. A collection of coins is made up of an equal number of pennies, nickels, dimes, and quarters. What is the largest possible value of the collection which is less than \$2?
- A. \$1.64 B. \$1.78 C. \$1.86 D. \$1.89 E. \$1.99
3. When the polynomial $P(x)$ is divided by $(x-2)^2$, the remainder is $3x - 3$. What is the remainder when $(x-1)P(x)$ is divided by $(x-1)(x-2)^2$?
- A. $3x - 3$ B. $3x^2 - 6x + 3$ C. 3 D. $x - 1$ E. $x - 2$
4. If $f(x) = 3x - 2$, find $f(f(f(3)))$. A. 19 B. 55 C. 75 D. 107 E. 163
5. What is the remainder when $x^3 - 2x^2 + 4$ is divided by $x + 2$?
- A. -12 B. 0 C. 4 D. 6 E. 12
6. Let p be a prime number and k an integer such that $x^2 + kx + p = 0$ has two positive integer solutions. What is the value of $k + p$?
- A. 1 B. -1 C. 0 D. 2 E. -2
7. What is the least number of prime numbers (not necessarily different) that 3185 must be multiplied by so that the product is a perfect cube?
- A. 1 B. 2 C. 3 D. 4 E. 5
8. Two adjacent faces of a three-dimensional rectangular box have areas 24 and 36. If the length, width, and height of the box are all integers, how many different volumes are possible for the box?
- A. 2 B. 3 C. 4 D. 5 E. 6
9. $(\tan t - \sin t \cos t)/(\tan t) =$
- A. $\sin t$ B. $\cos t$ C. $\sin^2 t$ D. $\cos^2 t$ E. 1
10. The counting numbers are written in the pattern at the right. Find the middle number of the 40th row.
- | | | | | | | | |
|--|----|----|----|----|----|----|----|
| | | | 1 | | | | |
| | | | 2 | 3 | 4 | | |
| | | 5 | 6 | 7 | 8 | 9 | |
| | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
- A. 1561 B. 1641 C. 1559 D. 1639 E. 1483
11. The solution set of $x^2 - 3x - 18 \geq 0$ is a subset of the solution set of which of the following inequalities?
- A. $x^2 - x - 20 \geq 0$ B. $(x - 4)/(x + 3) \geq 0$ C. $x^2 - 8x + 14 \geq 0$
D. both B and C E. all of A, B, and C

12. If $2a - 4b = 128b^3 - 16a^3$ and $a \neq 2b$, find $a^2 + 2ab + 4b^2$.

- A. $-1/8$ B. $-1/2$ C. $1/2$ D. $1/8$ E. 2

13. Square ABCD is inscribed in circle O (that is, A, B, C, and D all lie on the circle) and its area is a . Square EFGH is inscribed in a semicircle of circle O (that is, E and F lie on a diameter and G and H lie on the circle). What is the area of square EFGH?

- A. $a/5$ B. $2a/5$ C. $a/3$ D. $a/2$ E. $3a/5$

14. Consider all arrangements of the letters AMATYC with either the A's together or the A's on the ends. What fraction of all possible such arrangements satisfies these conditions?

- A. $1/5$ B. $2/15$ C. $1/3$ D. $2/5$ E. $3/5$

15. The year 2003 is prime, but its reversal, 3002, is not. In fact, 3002 is the product of exactly three different primes. Let N be the sum of these three primes. How many other positive integers are the products of exactly three different primes with this sum N ?

- A. 0 B. 1 C. 2 D. 3 E. 4

16. In a group of 30 students, 25 are taking math, 22 English, and 19 history. If the largest and smallest number who could be taking all three courses are M and m respectively, find $M + m$.

- A. 17 B. 19 C. 22 D. 23 E. 25

17. A boat with an ill passenger is $7\frac{1}{2}$ mi north of a straight coastline which runs east and west. A hospital on the coast is 60 miles from the point on shore south of the boat. If the boat starts toward shore at 15 mph at the same time an ambulance leaves the hospital at 60 mph and meets the ambulance, what is the total distance (to the nearest 0.5 mile) traveled by the boat and the ambulance?

- A. 60.5 B. 61 C. 61.5 D. 62 E. 62.5

18. If each letter in the equation $\sqrt{AMATYC} = MYM$ represents a different decimal digit, find T 's value.

- A. 3 B. 4 C. 5 D. 6 E. 7

19. If a , b , c , and d are nonzero numbers such that c and d are solutions of $x^2 + ax + b = 0$ and a and b are solutions of $x^2 + cx + d = 0$, find $a + b + c + d$.

- A. -2 B. -1 C. 0 D. 1 E. 2

20. Al and Bob are at opposite ends of a diameter of a silo in the shape of a tall right circular cylinder with radius 150 ft. Al is due west of Bob. Al begins walking along the edge of the silo at 6 ft per second at the same moment that Bob begins to walk due east at the same speed. The value closest to the time in seconds when Al first can see Bob is

- A. 46 B. 47 C. 48 D. 49 E. 50

- C 1. M has equation $-ax + by = c$, and N has equation $ax - by = c$. The x-intercepts are $-c/a$ and c/a , the y-intercepts are c/b and $-c/b$, and the slopes are both a/b , so only C is true.
- A 2. One penny, nickel, dime, and quarter are worth 41¢. The largest multiple of 41 less than 200 is 164.
- B 3. By the Division Algorithm, $P(x) = Q(x)(x - 2)^2 + (3x - 3)$, where $Q(x)$ is the quotient. Then $(x - 1)P(x) = Q(x)(x - 1)(x - 2)^2 + (x - 1)(3x - 3)$. Thus the new remainder is $3x^2 - 6x + 3$.
- B 4. Since $f(3) = 7$, $f(7) = 19$, and $f(19) = 55$, the answer is B.
- A 5. By the Remainder Theorem, the remainder is $(-2)^3 - 2(-2)^2 + 4$, or -12 .
- B 6. For p prime and positive solutions, the quadratic must factor into $(x - 1)(x - p)$, so that $k = -(p + 1)$. Thus the sum of k and p is -1 .
- E 7. The number 3185 factors into $(5)(7)(7)(13)$. To make it a perfect cube, multiply by 5, 5, 7, 13, and 13.
- E 8. The common edge must be a factor of both 24 and 36. Thus the possible dimensions of the box are $1 \times 24 \times 36$, $2 \times 12 \times 18$, $3 \times 8 \times 12$, $4 \times 6 \times 9$, $4 \times 6 \times 6$, and $2 \times 3 \times 12$. All six volumes are different.
- C 9. The given fraction equals $1 - \sin t \cos t \cot t = 1 - \cos^2 t = \sin^2 t$.
- A 10. Notice that n^2 is at the end of row n , so the middle term of row 40 must be $((39^2 + 1) + 40^2)/2 = 1561$.
- C 11. The given inequality has solution $(-\infty, -3] \cup [6, +\infty)$. But -3 does not satisfy either A or B (because of division by zero), so the answer must be C. Note that the solution of C is $(-\infty, 4 - \sqrt{2}] \cup [4 + \sqrt{2}, +\infty)$.
- A 12. Since $128b^3 - 16a^3 = 2(4b - 2a)(16b^2 + 8ab + 4a^2)$, then $2a - 4b = 2(4b - 2a)(16b^2 + 8ab + 4a^2)$, and $-1 = 8(a^2 + 2ab + 4b^2)$, so $a^2 + 2ab + 4b^2 = -1/8$.
- B 13. Let s be the side length of square EFGH. Then $s^2 + (s/2)^2 = r^2$, so $r = \frac{\sqrt{5}}{2}s$. Then a side of square ABCD $= 2r/\sqrt{2} = \frac{\sqrt{10}}{2}s$, and the area of ABCD $= a = (5/2)s^2$, so the area of EFGH $= s^2 = (2/5)a$.
- D 14. The number of arrangements of AMATYC is $6!/2 = 360$ (division by 2 accounts for the duplicate A's). If the A's are on the ends, there are $4! = 24$ ways to place the remaining 4 letters, and if the A's are together, there are 5 ways to place the A's (consider them as a single object) and $4!$ ways to place the remaining letters for a total of 120 arrangements. The required probability is $(24 + 120)/360 = 2/5$.
- C 15. Factoring 3002 yields $2(19)(79)$, so the prime factors add to 100. For three primes to add to 100, one must be a 2, and the only other possibilities for 98 are $31 + 67$ and $37 + 61$.
- E 16. Clearly $M = 19$. To find m , assume the math and English students overlap as little as possible. Since there are 5 students not taking math, the smallest overlap is 17. Then if the history students overlap the 8 math students not taking English and the 5 English students not taking math, there must be $19 - 5 - 8 = 6$ history students also taking both math and English. Thus $m = 6$ and $M + m = 25$.
- E 17. Let x be the distance from the hospital to the meeting point. Then $x/60 = \frac{\sqrt{7.5^2 + (60 - x)^2}}{15}$ (using time is distance over speed). This becomes $(x - 50)(x - 78) = 0$, whose only valid solution is 50. The boat then travels 12.5 miles for a total for the two vehicles of 62.5 miles.
- E 18. Examining the equation $AMATYC = MYM^2$ shows $316 < \sqrt{100000} \leq MYM \leq \sqrt{999999} < 999$. A quick check of the numbers 323, 343, 353, etc. shows $MYM = 363$ and $AMATYC = 131769$, so $T = 7$.
- A 19. The given conditions mean that $x^2 + ax + b = x^2 - (c + d)x + cd$, and $x^2 + cx + d = x^2 - (a + b)x + ab$. Thus $-a = c + d$, $b = cd$, $-c = a + b$, and $d = ab$. Hence $b = abc$ and $ac = 1$. Also, $a + c + d = 0 = a + b + c$, so $b = d$, which means $c = a = 1$. This means that $b = d = -2$, so $a + b + c + d = -2$.
- C 20. After t sec, Al's central angle is $6t/150$ rad, while Bob's distance from the silo is $6t$. Al first sees Bob when his location is the point of tangency of the line to Bob's position. This forms a right triangle, so that $\cos(\pi - 6t/150) = 150/(150 + 6t)$. This equation is satisfied by $t = 48.0075$.