

- A 1. Letting $t = \text{this}$ and $T = \text{that}$ yields the equations $(t + T)/3 = t^2$ and $T/t = 8/1$. Substituting $T = 8t$ into the first equation yields $3t^2 - 9t = 0$ or $3t(t - 3) = 0$, with solutions $t = 0, 3$. But $t \neq 0$, so $t = 3$.
- A 2. S is (2,3) and T is (-2,-3), so the required line has equation $y - 3 = \frac{3}{2}(x - 2)$, or $y = \frac{3}{2}x$.
- E 3. The given inequality is equivalent to $-2001 \leq 5 - 2x \leq 2001$, or $1003 \geq x \geq -998$. The resulting interval, $[-998, 1003]$, is contained only in the interval $[-1000, 2001]$.
- B 4. Let x be the given angle and y its supplement. $\sin x = \frac{2}{3}$ implies $\sin y = \frac{2}{3}$ and $\csc y = \frac{3}{2}$. Since $\cot^2 y + 1 = \csc^2 y$, then $\cot^2 y = \frac{5}{4}$ and $\cot y = \pm \frac{\sqrt{5}}{2}$. But y is in Quadrant II, so $\cot y = -\frac{\sqrt{5}}{2}$.
- C 5. The given function values yield the equations $c = 3$, $4a + 2b + c = 0$, and $16a + 4b + c = 1$. This implies $4a + 2b = -3$ and $16a + 4b = -2$, so $4b = -10$ and $b = -2.5$. Then $a = 0.5$, so $a + b + c = 1$.
- E 6. Since $Q(x) = \frac{(x-2)(x-1)}{x^2(x+1) - 4(x+1)} = \frac{(x-2)(x-1)}{(x-2)(x+2)(x+1)}$, the domain is all x except ± 2 and -1 .
- B 7. Since $8^2 + 15^2 = 17^2$, the triangle is a right triangle. Any right triangle inscribed in a circle must be inscribed in a semicircle with the hypotenuse as the diameter. Thus the radius is half of 17 or 8.5.
- D 8. The only 3-digit perfect cube with its first and last digits the same is 343, and the only 3-digit perfect squares divisible by 12 are 144, 324, 576, and 900, so $TYC = 576$. Thus $A + M + A + T + Y + C = 28$.
- E 9. For $2x^2 - bx - 36$ to have rational solutions, its discriminant, $b^2 + 288$, must be a perfect square. This is true for $b = \pm 1, \pm 6, \pm 14, \pm 21, \pm 34$, and ± 71 , for a total of 12 values of b .
- D 10. Since $\angle A = \angle CDA$, $\triangle ACD$ is isosceles with $AC = CD$. Then $AB = 3 = AC/3$, and since $\triangle ABF$ and $\triangle ACD$ are similar, $BF = BE/3 = CD/3 = 3$, and $EF = BE - BF = 6$.
- D 11. At 4 mph, Matt takes $(60 \text{ min})((1/2)/4) = 7.5 \text{ min}$ to row to shore. This means $(7.5 \text{ min})(10 \text{ gal/min}) = 75 \text{ gal}$ of water enter. Cassie must bail 45 gal (all but 30 of the 75 gal) in this 7.5 min, or 6 gal/min.
- D 12. If r is the length of the radius of the circle, the radius ending at the arc's midpoint, the chord, and the radius ending at one end of the chord form a right triangle with legs of length $r - 4$ and 6 mm and hypotenuse r mm. Then $(r - 4)^2 + 6^2 = r^2$, so $r^2 - 8r + 52 = r^2$, $8r = 52$, and the radius is exactly 6.5 mm.
- C 13. There are 5 ways to parenthesize: $(a + b)(c^a d)$, $((a + b)c)^a d$, $(a + (b * c))^a d$, $a + ((b * c)^a d)$, $a + (b * (c^a d))$. Evaluating each expression yields 7, 49, 1, 19, and 7, for 4 distinct values.
- B 14. We need the smallest N ending in 0, its digits adding to 8 or 17 (so that $N + 1$ is a multiple of 9), and $N + 2$ is a multiple of 8. The values satisfying the first two conditions are $80 + 90k$. Since 350 is the smallest value for which $N + 2$ is a multiple of 8 ($352 = 8 \times 44$), this is N . Only 11 is a factor of 352.
- A 15. The line segments $\overline{PS}$, $\overline{TW}$, $\overline{QV}$, $\overline{RU}$ divide the octagon into 4 isosceles right triangles, 4 rectangles, and a square. If s is the length of a side of the octagon, the total area is $4(s^2/4) + 4(s^2/\sqrt{2}) + s^2 = (2 + 2\sqrt{2})s^2$. The area of $PSTW = 2(s^2/\sqrt{2}) + s^2 = (1 + \sqrt{2})s^2 = \text{half the octagon's area}$.
- E 16. The equation is equivalent to $x^3 + y^3 - x - y = 0$, or $(x + y)(x^2 - xy + y^2 - 1) = 0$. Thus $x + y = 0$ (a straight line), or $x^2 - xy + y^2 = 1$, an ellipse (since $B^2 - 4AC < 0$).
- C 17. To maximize the product, the factors should be as nearly equal as possible. For $n = 2, 3, 4, 5, 6$, this yields $5(5)$, $3(3)(4)$, $2(2)(3)(3)$, $2(2)(2)(2)(2)$, and $1(1)(2)(2)(2)(2)$, or 25, 36, 36, 32, 16.
- B 18. The number of ways to pick any 2 shoes is $C(22, 2) = 231$. The number of ways to pick a matching pair is by picking 1 right and 1 left of the same color: $6(6) + 3(3) + 2(2) = 49$. Then $49/231 = 7/33$.
- C 19. If A is (0,0) and B is (65,0), the lines thru A, C and B, E have equations $y = \frac{3}{4}x$ and $y = \frac{-5}{12}x + \frac{325}{12}$. They intersect at $(\frac{325}{14}, \frac{975}{56})$, and the pentagon's area is $\frac{52(39)}{2} + \frac{25(60)}{2} - \frac{975}{56}(\frac{65}{2}) = 1198 \frac{17}{112}$.
- D 20. By symmetry they meet after 6 turns. Starting from (0,0) they end at $(\cos 0^\circ + \cos 15^\circ + \cos 30^\circ + \dots + \cos 75^\circ + \cos 90^\circ, \sin 0^\circ + \sin 15^\circ + \dots + \sin 90^\circ)$, which is $4.30\sqrt{2}$ or about 6.1 from (0,0).

- E 1. $x^2 - 15x - 54 = (x - 18)(x + 3)$. No equation is satisfied by 18, and -3 satisfies only $6x + 15 = 2x + 3$.
- B 2. For an angle a , $180 - a = k(90 - a)$ or $a = \frac{90k - 180}{k - 1}$. If $k = 2$, $a = 0$, so $k > 2$. If $k = 3$, $a = 45^\circ$.
- D 3. Since hypotenuse PR in right $\triangle PQR$ is twice leg PQ , $\triangle PQR$ is a 30-60 right triangle with $\angle QRP = 30^\circ$ and $QR = \sqrt{3}$. The same argument shows $\angle SRP = 30^\circ$, so $\triangle SQR$ is equilateral and $SQ = QR = \sqrt{3}$.
- D 4. $16^{-2} = .00390625$ and $16^{-2.25} = .001953125$, so $\log_{16} .002$ is closer to -2.5 than any other choice.
- E 5. The parallelogram which is their intersection has base the hypotenuse of a 30-60 right triangle with longer leg 60 ft, and altitude 50 ft. The base is then $\frac{120}{\sqrt{3}}$ ft and the area is $\frac{120(50)}{\sqrt{3}}$ or about 3464 sq ft.
- A 6. The larger kitten must weigh half of 12 lb (since its weight equals the other 2), the smaller kitten must weigh a third of 12 lb (since its weight is half the other 2), so the puppy must weigh $12 - 6 - 4 = 2$ lb.
- B 7. If R is the final result, $(R - 2) + (R + 2) + (R/2) + 2R = 45$ and $R = 45(2/9) = 10$. The numbers are then 8, 12, 5, and 20 with a product of 9600.
- D 8. Since $2002 = 2(7)(11)(13)$, its positive integer factors are 1, 2, 7, 11, 13, $2(7)$, $2(11)$, $2(13)$, $7(11)$, $7(13)$, $11(13)$, $2(7)(11)$, $2(7)(13)$, $2(11)(13)$, $7(11)(13)$. The palindromes are 1, 2, 7, 11, 22, 77, 1001.
- B 9. Draw the diameter of the larger circle which passes through the center of the smaller circle. The required segment goes from the larger to the smaller circle along this diameter and has length $b - a - d$.
- A 10. The set can be rewritten as $\{24^{1000}, 28^{1000}, 32^{1000}, 27^{1000}, 25^{1000}\}$, so the first one is smallest.
- C 11. There is one ending 0 in $n!$ for every factor of 5 in every number $\leq n$, so the number of ending zeros for $n!$ is $n/5 + n/25 + n/125 + \dots$ (all rounded down). A little trial and error shows $n = 360$.
- A 12. Solving the equations $p + j + s = 60$ and $30p + 80j + 120s = 4400$ simultaneously yields $p = 8 + 0.8s$ and $j = 52 - 1.8s$. If $j \geq 0$, $s \leq 260/9$, so $s \leq 28$. But 25 is the largest value of s for which p is an integer.
- C 13. Since $\cos 3x \cos x - \sin 3x \sin x = \cos(3x + x)$, the equation is equivalent to $\cos 4x = 0$. Thus $4x$ is an odd multiple of $\pi/2$, and x is an odd multiple of $\pi/8$. The interval $[-8\pi, 8\pi]$ has 64 such multiples.
- D 14. If the triangle's legs are s and $\sqrt{K}$, then $s + \sqrt{K} + \sqrt{s^2 + K} = K$ and $(1/2)s\sqrt{K} = K$. But this implies $s = 2\sqrt{K}$. Substitution yields $3\sqrt{K} + \sqrt{5K} = K$, so $K = (3 + \sqrt{5})^2 = 14 + 6\sqrt{5}$, and $a + b + c = 25$.
- D 15. To leave a remainder of 1 on division by 2, 3, 4, 5, and 6, the number must be 1 more than a multiple of 60. The smallest such number divisible by 7 is 301, whose largest prime factor is 43.
- E 16. There are $10(9)(8) = 720$ ways to select 3 different digits, but only $1/6$ of them, or 120, have nondecreasing order. There are 90 ways to select 2 different digits, with half having nondecreasing order. The digits a, b produce 2 sequences, a, a, b or a, b, b , for 90 more. Finally, there are 10 sequences a, a, a .
- C 17. Since $AMA > TYC$ and $AMA - TYC < 10$, $A = T + 1$. Also, $M = 0$ and $Y = 9$ (otherwise, the difference ≥ 10). The uniqueness condition eliminates $AMA = 808, 707, 606$, and the only solutions are $505 - 498$ or $497, 404 - 398$ or $396, 303 - 298, 297, 296$, or 295 , and $202 - 198, 197, 195$, or 194 .
- B 18. $\triangle DCF$ is similar to each of the 3 small triangles formed by the intersection of the 2 rectangles. Solving a few similarity proportions shows the required region to be a trapezoid with height 12 and bases 40 and 15. Its area is thus $12(40 + 15)/2$ or 330.
- B 19. The required region is an isosceles right triangle with hypotenuse 20 ft, a three-quarter circle with radius 20 ft, and two 135° sectors with radii $20 - 10\sqrt{2}$ ft (forming another three-quarter circle). The total area is $\frac{1}{2}(10\sqrt{2})^2 + \frac{3}{4}(400\pi) + \frac{3}{4}\pi(20 - 10\sqrt{2})^2 = 100 + 750\pi - 300\pi\sqrt{2}$, or about 1123 sq ft.
- E 20. $P(\text{the numbers are different}) = 8/9$ (since for each first number, there are 8 different second numbers). $P(\text{the numbers are different if they add to } k) = 1$, if k is odd (since two numbers with odd sum are never equal), and $(k-2)/(k-1)$ if k is even (since of the $k-1$ such sums, exactly one of them has two equal values). Independence means the two probabilities are equal, but this cannot happen if the second one is 1; thus k is even, and $(k-2)/(k-1) = 8/9$, meaning $k = 10$.

- 3 1. $\log_{36} 9 + \log_{36} 24 = \log_{36} 216$, so $216 = 36^x$, $6^3 = 6^{2x}$, and $x = 1.5$.
- D 2. The angle sum in a regular 30-sided polygon is $(30 - 2)180^\circ = 5040^\circ$, so each angle is $5040/30 = 168^\circ$.
- A 3. The first system's solution is $(4, -1)$, so $p = 4$, $q = -1$. The other system's solution is $(4, 1)$, or $(p, -q)$.
- C 4. Since $y = (c/360)(2\pi r) = (k/360)(2\pi R)$, then $cr = kR$, and $c/k = R/r$.
- C 5. The angles are either $k + 4k + 4k = 180$, so $k = 20^\circ$ and $4k = 80^\circ$, or $k + k + 4k = 180$, so $k = 30^\circ$ and $4k = 120^\circ$. Thus the only choice which is not possible is 40° .
- D 6. $\frac{2}{15} = \frac{1}{8} + \frac{1}{120} = \frac{1}{9} + \frac{1}{45} = \frac{1}{10} + \frac{1}{30} = \frac{1}{12} + \frac{1}{20}$. These are the only solutions for $8 \leq a \leq 14$, and thus no other solutions are possible, since $1/7 > 2/15$, and $1/a$ must be more than half of $2/15$.
- D 7. The smallest value of k is at least $\text{GCF}(36, 21) = 3$. Trial-and-error shows that $(3, -5)$ is a solution when $k = 3$, and all solutions have the form $(3 + 7i, -5 - 12i)$. Thus the smallest positive value for y is 7.
- D 8. Since tangent and cotangent are cofunctions, $\cot x = \tan(\pi/2 - x) = \tan(-x + \pi/2)$, which is a reflection in the x -axis followed by a translation $\pi/2$ units to either the left or right (since period of tangent is π).
- E 9. The x -intercepts of $T(x) = \cos x^{-2}$ on $[0.05, 1]$ are the same as those of $t(x) = T(x^{-0.5}) = \cos x$ on $[1, 400]$. The x -intercepts of $t(x)$ are at $\pi/2 + \pi k$ and $\pi/2 + 126\pi = 397.411$, so the answer is 127.
- E 10. Let $h : m$ be the required time. The minute hand moves 6° per minute ($360^\circ/60$), so its position is $6m$. The hour hand starts at $30h$ ($360^\circ/12$) and moves $1/2^\circ$ per minute ($30^\circ/60$), so its position is $30h + 0.5m$. Then $|30h + 0.5m - 6m| = m/2$, so $m = 5h$ or $6h$. If $m = 5h$, h must be even to get a whole number angle, so the times are 2:10, 4:20, 6:30, 8:40, 10:50, 1:06, 2:12, 3:18, 4:24, 5:30, 6:36, 7:42, 8:48, and 9:54.
- B 11. For $AM \cdot A$ to be 600 to 699, A must be 8 ($A = 7$ makes $T \leq 5$, $A = 9$ makes $T \geq 8$). Trying 80 up to 87 (88 and above makes $T = 7$) and avoiding repetition produces the unique solution $84 \cdot 8 = 672$.
- B 12. A year is 52 weeks + 1 day, so dates shift 1 day later each year (2 in leap years), and 27 paychecks means getting one on Jan. 1. Juan gets paychecks on Jan. 6, 2003, Jan. 5, 2004, Jan. 3, 2005 (because 2004 is a leap year), Jan. 2, 2006, and Jan. 1, 2007, with 27 paychecks that year.
- C 13. $M^2 = \begin{bmatrix} s^2 - 8 & 2s + 2v \\ -4s - 4v & v^2 - 8 \end{bmatrix}$, so $s = \pm\sqrt{8}$, $v = -s$, and $|s - v| = |2s| = 4\sqrt{2}$.
- A 14. Since $2002 = 2(7)(11)(13)$, $N = 2002$. The product of 2002's factors is 2002^8 , since the 16 factors can be paired up; but if P is the common product, the 16 factors multiply to P^4 , so $P = 2002^2$. That means each row, column, and diagonal has 2 factors each of 2, 7, 11, and 13. By process of elimination, $\# = 22$.
- B 15. The allowed values for b & c are: $0:0, -1, -4, -9$; $\pm 1:0, -2, -6$; $\pm 2:1, 0, -3, -8$; $\pm 3:2, 0, -4, -10$; $\pm 4:3, 0, -5$; $\pm 5:4, 0, -6$; $\pm 6:5, 0, -7$; $\pm 7:6, 0, -8$; $\pm 8:7, 0, -9$; $\pm 9:8, 0, -10$; $\pm 10:9, 0$. The answer is $76/441 \approx 0.17$.
- E 16. Label the functions f_1, f_2, f_3 . Since $f_1(0) = f_2(0)$, eliminate A; $f_1(1) = f_2(1) = f_3(1)$ eliminates B; $f_1(-1) = f_3(-1)$ eliminates C. Solving $f_2(x) - f_3(x) = 0$ gives $x = (c - a)/(b - c)$; this eliminates D.
- A 17. The first several terms have the pattern $s_n^2 - s_{n-1}s_{n+1} = 5$ for odd n and -5 for even n . Assuming this pattern holds up to 2001, $s_{2002}^2 - s_{2001}s_{2003} = s_{2002}^2 - s_{2001}(s_{2001} + s_{2002}) = s_{2002}^2 - s_{2001}^2 - s_{2001}s_{2002} = s_{2002}^2 - (s_{2000}s_{2002} + 5) - s_{2001}s_{2002} = s_{2002}(s_{2002} - (s_{2000} + s_{2001})) - 5 = s_{2002}(0) - 5 = -5$.
- B 18. For $x \leq -1/2$, $f(x) = -1$; for $-1/2 \leq x \leq 1/2$, $f(x) = 2x$; for $x \geq 1/2$, $f(x) = 1$. Thus $f(f(x)) = 4x$ for $-1/4 \leq x \leq 1/4$, and $f(f(f(x))) = 8x$ for $-1/8 \leq x \leq 1/8$. The required interval has length $1/8 - (-1/8) = 1/4$.
- A 19. Because $k(x) = (x - p)(x - q)(x - r) = x^3 - (p + q + r)x^2 + (pq + qr + pr)x - pqr$, then $p + q + r = 5$, $pq + qr + pr = 4$, and $pqr = -8$. But $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{qr + pr + pq}{pqr} = \frac{4}{-8} = -\frac{1}{2}$.
- A 20. By polynomial division, $P(x) = Q(x)(x^3 + x + 1) + (x - 2)$. Any root of both $P(x)$ and $Q(x)$ must also then be a root of $x - 2$. But the only root of $x - 2$ is 2, and $x^2 + ax + 1$ has a root of 2 only if $a = -5/2$.

$2x^2 - rx + 10 = 0$) can only have one solution. This happens only if $b^2 - 4ac = 0$, or $r^2 - 80 = 0$. Thus $r = \pm\sqrt{80} = \pm 4\sqrt{5}$. Only the positive solution gives a point of tangency in Quadrant I.

B 20. Since the base of the triangle is fixed, the maximum area occurs when C is the highest point of the circle, and the minimum area occurs when C is the lowest point on the circle. These extrema occur when C is (0,3) and (0,1), giving areas of $3(1)/2$ or 1.5 and $1(1)/2$ or 0.5, for a product of 0.75.