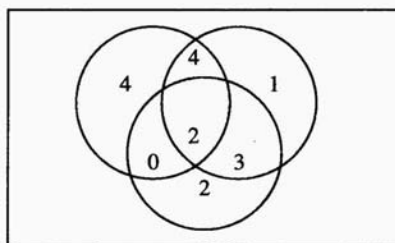


1. A stock loses 60% of its value. What must the percent of increase be to recover all of its lost value?
A. 60% B. 120% C. 150% D. 200% E. 300%
2. Which of the following is NOT a factor of $x^4 - 4x^3 - x^2 + 16x - 12$?
A. $x - 2$ B. $x + 2$ C. $x - 1$ D. $x + 1$ E. $x - 3$
3. The library in Johnson City has between 1000 and 2000 books. Of these, 25% are fiction, $1/13$ are biographies, and $1/17$ are atlases. How many books are either biographies or atlases?
A. 240 B. 250 C. 270 D. 280 E. 300
4. A trecimal is like a decimal, except the digits represent fractions with powers of 3 instead of 10. For example, $16/27 = 1/3 + 2/9 + 1/27 = 0.121$ as a trecimal. How is $77/81$ expressed as a trecimal?
A. 0.950617 B. 0.2012 C. 0.1211 D. 0.1111 E. 0.2212
5. The function $P(t) = \cos 8t$ can be written as sums and differences of powers of $\cos t$. When $P(t)$ is written that way, what is the coefficient of $(\cos t)^3$?
A. 0 B. 1 C. -1 D. 2 E. -2
6. If $\log_a b = 64$, find $\log_{a^2} b^3$.
A. 16 B. 48 C. $128/3$ D. 96 E. 512
7. The number $877530p765q6$ is divisible by both 8 and 11, with p and q both digits from 0 to 9. The number is also divisible by
A. 7 B. 12 C. 16 D. 18 E. not enough information to know
8. Teams A and B play a series of games; whoever wins two games first wins the series. If Team A has a 70% chance of winning any single game, what is the probability that Team A wins the series?
A. 0.616 B. 0.637 C. 0.657 D. 0.700 E. 0.784
9. The Venn diagram at the right represents sets A, B and C (not necessarily in that order). Depending on how the diagram is labeled, how many different answers are possible for the number of elements in the set $A - B$? (Note: $A - B$ is all elements which are in A but not in B)
A. 2 B. 3 C. 4 D. 5 E. 6



10. A fixed point for a function $y = f(x)$ is a real number r such that $f(r) = r$. How many of the following functions must have a fixed point?

polynomial function of odd degree > 1
trigonometric function $y = A \sin Bx + D$

polynomial function of even degree > 0
rational function $y = (x - a)/(x - b)$

- A. 0 B. 1 C. 2 D. 3 E. 4

11. Which of the following is the identity function $f(x) = x$ for all real numbers?

- A. $e^{\ln x}$ B. $\ln e^x$ C. $\sin(\arcsin x)$ D. $\arctan(\tan x)$ E. $\sqrt{x^2}$

12. A circular table is pushed into a corner of a rectangular room so that it touches both walls. A point on the edge of the table between the two points of contact is 2 inches from one wall and 9 inches from the other wall. What is the radius of the table?
- A. 5 inches B. 12 inches C. 15 inches D. 17 inches E. 20 inches
13. In $\triangle ABC$, $\angle C = 90^\circ$ and $\cos \angle A = 4/5$. If D is the midpoint of side AC, find $\cos \angle CDB$.
- A. $\frac{2\sqrt{13}}{13}$ B. $\frac{5}{9}$ C. $\frac{\sqrt{5}}{4}$ D. $\frac{2}{5}$ E. $\frac{3}{5}$
14. Enrique walks along a level road and then up a hill. At the top he immediately turns and walks back to his starting point. He walks 4 mph on level ground, 3 mph uphill, and 6 mph downhill. If the entire walk takes 6 hours, how far does he walk?
- A. 16 mi B. 20 mi C. 24 mi D. 28 mi E. 32 mi
15. If $x^2 = x + 3$, then $x^3 =$
- A. $x + 6$ B. $4x + 3$ C. $4x^2 + 3$ D. $x^2 + 3x + 3$ E. $x^2 + 27$
16. A bag holds 5 cards identical except for color. Two are red on both sides, two are black on both sides, and one is red on one side and black on the other. If you pick a card at random and see that the only side you can see is red, what is the probability that the other side is also red?
- A. $1/2$ B. $2/3$ C. $3/4$ D. $4/5$ E. $5/6$
17. The set S contains the number 2, and if it contains the number n, it also contains $3n$ and $n + 5$ (assume S contains only numbers produced by these rules). Which of the following is NOT in S?
- A. 2000 B. 2001 C. 2002 D. 2003 E. 2004
18. Let $f(x) = ax + b$, with $b < a$ both positive integers. If for positive integers p and q, $f(p) = 18$ and $f(q) = 39$, what is the value of b?
- A. 1 B. 3 C. 4 D. 7 E. 8
19. In $\triangle SBC$, $SB = 12$, $BC = 15$, and $SC = 18$. Let O be the point for which BO bisects angle SBC and CO bisects angle SCB. If M and L are on sides SB and SC respectively so that ML is parallel to side BC and contains point O, what is the perimeter of $\triangle SML$?
- A. 24 B. 27 C. 30 D. 32 E. 36
20. Ed has four children, Al, Bo, Cy, and Di (in order oldest to youngest). Bo is 4 years older than Cy and 12 years older than Di. This year Ed notices that he is twice as old as Bo, and the sum of the squares of the children's ages equals the square of Ed's age. If Di just became a math teacher, what is the sum of the children's ages?
- A. 48 B. 76 C. 100 D. 128 E. 148

Test #2

Student Mathematics League Short Solutions

February-March 2004

- C 1. Suppose the original value was \$100. A 60% loss reduces it to \$40. An increase of \$60 is 150%.
- D 2. The expression $x^4 - 4x^3 - x^2 + 16x - 12 = (x^4 - 4x^3 + 4x^2) - (5x^2 - 16x + 12) = x^2(x - 2)^2 - (5x - 6)(x - 2) = (x - 2)(x^2(x - 2) - 5x + 6) = (x - 2)(x(x - 1)^2 - 6(x - 1)) = (x - 2)(x - 1)(x - 3)(x + 2)$.
- A 3. The number of books in the library must be divisible by 4, 13, and 17. Thus the number of books is a multiple of $(4)(13)(17) = 884$. The only multiple of 884 between 1000 and 2000 is 1768, so the number of biographies or atlases must be $1768/13 + 1768/17 = 136 + 104 = 240$.
- E 4. The fraction $77/81 = 2/3 + 23/81 = 2/3 + 2/9 + 5/81 = 2/3 + 2/9 + 1/27 + 2/81 = 0.2212$.
- A 5. $\cos 8t = 2 \cos^2 4t - 1 = 2(2 \cos^2 2t - 1)^2 - 1 = 2(2(2 \cos^2 t - 1)^2 - 1)^2 - 1$, which involves only even powers of $\cos t$, so the coefficient of $(\cos t)^3$ is 0.
- D 6. If $\log_a b = 64$, then $a^{64} = b$, $(a^2)^{32} = b$, and $(a^2)^{96} = b^3$. Thus $\log_{a^2} b^3 = 96$.
- B 7. A number is divisible by 8 only if its last three digits are, so q must be 3 or 7. It is easy to determine that q cannot equal 7, so $p = 3$ and $q = 3$. The resulting number is divisible by 3, but not by 7, 9, or 16.
- E 8. Team A wins the series if it wins the first two games or if it wins exactly one of the first two games and then the third game. This probability is $(0.7)(0.7) + (0.7)(0.3)(0.7) + (0.3)(0.7)(0.7) = 0.784$.
- C 9. The number of elements in $A \cup B$ could be the sum of any two numbers inside one circle and outside another. These are $4 + 4$, $4 + 1$, $1 + 3$, $3 + 2$, $2 + 0$, and $0 + 4$, which yield four distinct sums.
- C 10. Graphically, a function has a fixed point if it intersects the line $y = x$. Since an odd degree polynomial approaches $+\infty$ in one direction and $-\infty$ in the other, it must cross this line. An even degree polynomial could lie entirely above the line; a sine curve must cross it; a rational function could have the line $y = x$ as an asymptote. Thus the answer is two.
- B 11. The function $e^{\ln x}$ is undefined for $x \leq 0$; $\sin(\arcsin x)$ is undefined for $|x| > 1$; $\arctan(\tan x) = x$ only if x is between $-\pi/2$ and $\pi/2$; $\sqrt{x^2} = |x|$, not x ; so the answer must be $\ln e^x$.
- D 12. Let r be the radius of the table; then $(r - 2)^2 + (r - 9)^2 = r^2$, since the given point is one vertex of a right triangle. Then $r^2 - 22r + 85 = (r - 5)(r - 17) = 0$, and $r > 9$ implies $r = 17$.
- A 13. The given triangle must have $AC = 4x$, $BC = 3x$, and $AB = 5x$. Then $DC = 2x$, $BD = x\sqrt{13}$, and $\cos \angle CDB = 2/\sqrt{13}$.
- C 14. Let L = the distance Enrique walks on the level and H = the distance he walks uphill. Then $L/4 + H/3 + H/6 = 6$ hr. But then $L + 2H = 24$, and $L + 2H$ is his total distance.
- B 15. If $x^2 = x + 3$, then $x(x^2) = x(x + 3) = x^2 + 3x = (x + 3) + 3x = 4x + 3$.
- D 16. There are 5 possible red sides. Of those sides, 4 have a red side on the other side, so the required probability is $4/5$.
- A 17. From the rules, if n is not a multiple of 5, then neither is $3n$ or $n + 5$. Since S starts out containing 2 (not a multiple of 5), S contains no multiples of 5, hence not 2000. For the other numbers, $2001 = 3(2 + 5(133))$, $2002 = 2 + 5(400)$, $2003 = 3(3(2 + 5(44))) + 5$, and $2004 = 3(3(3(2))) + 5(390)$.
- C 18. We know $ap + b = 18$ and $aq + b = 39$, so $a(q - p) = 21$. Thus $a = 1, 3, 7$, or 21 . But $a \neq 1$ (since then $b = 0$), $a \neq 21$ (since then $b < 0$), and $a \neq 3$ (since then $b = 0$). Thus $a = 7$, $b = 4$, $p = 2$, $q = 5$.
- C 19. By definition of bisect, $\angle OBM = \angle OBC$ and $\angle OCL = \angle OCB$. But $\angle OBC = \angle BOM$ and $\angle OCB = \angle COL$ (by alternate interior angles). Thus $\angle OBM = \angle BOM$ and $\angle OCL = \angle COL$, so triangles OBM and OCL are isosceles with $MB = MO$ and $LC = LO$. The perimeter of $\triangle SML = SB + SC = 30$.
- E 20. From the given information, $4B^2 = A^2 + B^2 + (B - 4)^2 + (B - 12)^2$, so $B^2 + 32B - 160 = A^2$. Completing the square yields $(B + 16)^2 - A^2 = 416$. The only positive integers satisfying this equation are $A = 103$, $B = 89$ (making Ed 178); $A = 50$, $B = 38$; $A = 22$, $B = 14$ (making Di 2 yrs old). Thus the only physically possible solution is: Al is 50, Bo is 38, Cy is 34, Di is 26, with total ages 148.