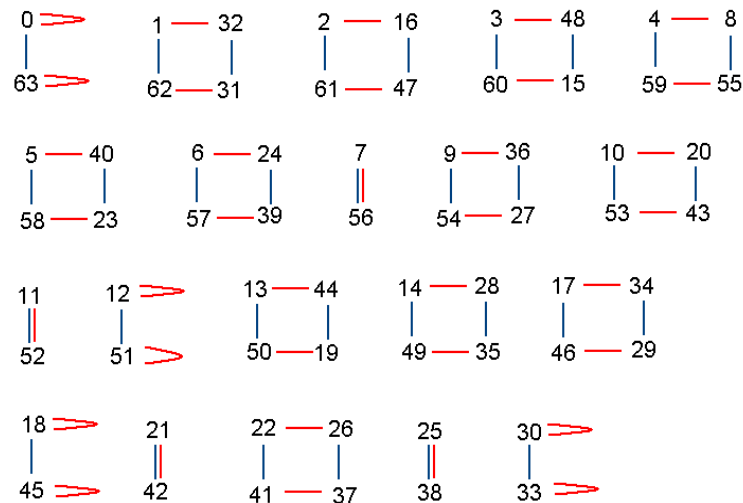


14. A binary string is a sequence of 1's and 0's, such as 10011 or 11101010. How many different binary strings of length 6 are there such that no two are reversals of each other or add up to 111111?

A. 22 B. 23 C. 24 D. 25 E. 26

Solution: There are $2^6 = 64$ six-long binary strings. We will count the complement and subtract it from 64. If we interpret these numbers as written in base 2, they are the numbers from 0 to 63. We can then pair them up so that the pairs add up to 111111 - which is the same as two numbers adding up to 63. So the pairs are 0 with 63, 1 with 62, and so on, 30 with 33, and finally 31 with 32. So if we select one from the pair into our collection, then we cannot select the other. How about the reversal? Each number has a unique number as their reversal. This is another number UNLESS the number is symmetrical and is therefore its own reversal. How many such symmetrical numbers are there? We claim 8. This is because we have complete freedom to select the first three digits - giving us 8 choice but then there is no choice but to duplicate the triple backwards to get a symmetrical string. For example 110 will give us the string 110011. So, out of the 64 strings 8 are symmetrical and so the other 56 can be paired into 28 pairs where they are each other's reversal. The question is: can we pick just one from each of the 28 pairs so that no two add up to 63?

Solution: The blue lines connect two numbers that add to 111111 and the red lines connect strings that are reversals of each other.



If we wanted to collect numbers in a set such that no two are connected, then

- we can pick one from each pairs such as 0-63 or 7-56
- we can pick exactly two from each of the squares. For example, the first square, containing 1-32-31-62, we can either select the pair 1 and 31 or the pair 32 and 62.

This means that a maximum of 32 such numbers can be collected.