1. Define the operation Ω by $\Omega(x, y, z) = \frac{2x^2+3y-5z}{x}$. Find $\Omega(1, \Omega(2,3,4), 5)$.

A. -55/2 B. -3/2 C. 0 D. 3/2 E. 55/2

2. How many pairs of positive integers a, b with $a < b$ have 24 as the least common multiple of a and b?

A. 8 B. 9 C. 10 D. 371 E. 372

3. Suppose circles C_1 and C_2 share center C but have radii 1 and 2, respectively. If the tangent to C_1 at a point P crosses C_2 at the point Q, and the line containing QC also crosses C_2 at the point R, determine the area of triangle PQR.

A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. $2\sqrt{3}$

4. There are two real values of b for which the function $f(x) = \frac{2x^3+bx^2+x}{x-3}$ can be re-written in the form $f(x) = 2(x-k)^2 + \frac{57+9b}{x-3}$ for some real number k. Find the sum of those two values of b.

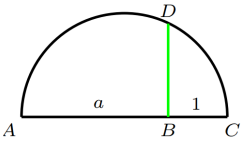
A. -38 B. -12 C. 0 D. 12 E. 38

5. Suppose a and b are positive integers, neither of which is divisible by 3, but whose sum is divisible by 3. Which of the following statements **must** be true?

I. $a^2 + b^2$ is divisible by 3
II. $a^2 - b^2$ is divisible by 3
III. $a^3 + b^3$ is divisible by 3

A. I only B. II only C. III only D. I and II only E. II and III only

6. In the given semicircle, AC is perpendicular to BD, $|AB| = a$, and $|BC| = 1$. Find $|BD|$.



A. a B. $\sqrt{a} + 1$ C. $\sqrt{a} - 1$ D. $a - 1$ E. $\sqrt{a}$

7. Let S be the square in the xy-plane with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$. Suppose S is rotated 45° counter-clockwise about the origin so that it has a highest vertex, and is then rotated a second time 45° clockwise about its highest vertex. What is the sum of the coordinates of the four vertices of the transformed square after both of these rotations?

A. $4 - 4\sqrt{2}$ B. $4\sqrt{2} - 4$ C. 4 D. $4\sqrt{2}$ E. $4 + 4\sqrt{2}$

8. Suppose $\log_a b = 64$. Find $\log_{a^2} b^3$.

A. 16 B. 48 C. 128/3 D. 96 E. 512

9. A gift exchange is planned for 7 people so that no person gives a gift to themself or to the person who gives them a gift. An exchange is different from another exchange if any person gives a gift to a different person. How many different ways can this exchange be carried out?

A. 740 B. 1140 C. 2040 D. 5033 E. 5040

10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). Each of them knows who is which and they also know that you know that there is one of each in the room. X says "Z is a spy.", Y says "X is a knave.", and lastly, Z says "X and Y gave you enough information to determine who is the knight." Who is the knave?

A. X B. Y C. Z D. Impossible to determine from given information

11. How many 5 digit numbers are there which are a multiple of 9, have no digit as a 0, and the middle digit is the sum of the other 4 digits?

A. 6 B. 20 C. 44 D. 56 E. 60

12. Suppose a triangle in the xy-plane has vertices $(0,0), (3,0), (0,4)$. Let R be the interior of the triangle and S be the set of points within 1 unit of this triangle's vertices or edges. What is the area of $S \cup R$?

A. $9+\pi$ B. $12+\pi$ C. $14+\pi$ D. $15+\pi$ E. $18+\pi$

13. Suppose f is a real valued function defined on the set of real numbers and that $f(x)+2f(2-x)=x^2$. What is the value of $f(3)$?

A. $-\frac{7}{3}$ B. $-\frac{5}{3}$ C. 0 D. $\frac{5}{3}$ E. $\frac{7}{3}$

14. The sequence of digits $1234567891011121314\ldots$ is obtained by writing the positive integers 1, 2, 3, ... one after another. Counting from the left, the 10^{th} digit is a 1, while the 15^{th} digit is a 2. What is the 2024^{th} digit?

A. 1 B. 3 C. 5 D. 7 E. 9

15. A triangular pyramid is formed from 2024 balls such that 1 ball is at the top layer, the second layer is comprised of 3 balls in the form of a triangle, the third layer is comprised of 6 balls in the form of a triangle, etc. Each layer is a triangular array of balls supporting all previous layers. How many layers does this pyramid have?

A. 18 B. 22 C. 24 D. 26 E. 28

16. How many of the following are divisible by 2024?

I. The number formed by repeating "2024" 23 times.
II. $(4!)!$
III. The product of the number formed by repeating the digit "4" 2023 times with the number formed by repeating the digits "46" 2023 times.

A. 0 B. 1 C. 2 D. 3 E. Impossible to determine

17. Let $S(N)$ = the sum of all remainders when an integer N is successively divided by 2, 3, 4, …, 10. For example, $S(322) = 0 + 1 + 2 + 2 + 4 + 0 + 2 + 7 + 2 = 20$. If $100 \le N < 1000$, find the minimum possible value of $S(N)$.

A. 2 B. 3 C. 4 D. 5 E. 6

18. The goal of baseball is to score more runs than the opponent. Note that baseball games cannot end in a tie, so at least one run must be scored by at least one team in every game. The "Pythagorean Theorem for Baseball" says that the expected winning percentage for a baseball team is given by $P=\frac{r^2}{r^2+a^2}$, where r is the total number of runs scored by the team, and a is the total number of runs allowed by that team. If the Southtown Skippers have played 3 games, which of the following is NOT a possible value of P?

A. 0 B. 4/13 C. 1/2 D. 2/3 E. 1

19. Determine the exact value of $\sec(\operatorname{arccot}(\tan(\arcsin(\cos(\arctan(15/8))))))$.

A. 8/17 B. 8/15 C. 15/8 D. 16/8 E. 17/8

20. In the given diagram, each variable p through x represents a different integer from 1 to 9. The operations in each row performed from left to right and in each column from top to bottom (not using order of operations) result in the numbers at the end of each row and the bottom of each column. Find the value of $p + t + x$.

A. 14 B. 16 C. 18 D. 21 E. 24

p	+	q	÷	r	4
+		+		+	
s	–	t	×	u	5
÷		÷		+	
v	+	w	×	x	42
4		3		13	