

The Title

The Author

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1. For real numbers a and b , define an operation $\blacksquare$ as $a \blacksquare b = ab^2 - |a|$. Find $[(-2) \blacksquare 5] \blacksquare (-1)$.

A. -104 B. -96 C. -53 D. 0 E. 96

Solution:

$$[(-2) \blacksquare 5] \blacksquare (-1) = ((-2) 5^2 - |-2|) \blacksquare (-1) = (-52) \blacksquare (-1) = -52(-1)^2 - 1|-52| = -104$$

2. Let $n = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ where a , b , and c are all positive integers. What is the largest possible value of n that is less than 1?

A. $\frac{9}{10}$ B. $\frac{11}{12}$ C. $\frac{19}{20}$ D. $\frac{41}{42}$ E. $\frac{63}{64}$

Solution: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

Try decreasing the smallest number.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{7} = \frac{41}{42} \quad \text{the answer is D or E yeay!}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{19}{20}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$$

Claim: We can't get $\frac{63}{64} = 1 - \frac{1}{64}$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{63}{64} \quad \text{if GCD is 64, then one of the denominators must be 64}$$

$$\frac{1}{64} + \frac{1}{a} + \frac{1}{b} = \frac{63}{64}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{31}{32} \quad \text{again, then } b = 32$$

$$\frac{1}{a} + \frac{1}{32} = \frac{31}{32} \quad \frac{15}{16} = \frac{1}{a} \quad \text{nope}$$

$$\frac{bc + ac + ab}{abc} = \frac{63}{64}$$

$$64(bc + ac + ab) = 63abc$$

64 is a divisor of abc

3. Michael is playing a game that involves two quarters (25 cents each), three dimes (10 cents each), one nickel (5 cents), and four pennies (1 cent each). He flips each of the coins once, and wins all of the coins that land heads up. If all of the coins are fair coins (the probability of landing heads up is $\frac{1}{2}$ and the probability of landing tails up is also $\frac{1}{2}$), what is the probability (to the nearest hundredth) that he will win at least fifty cents?

A. 0.40 B. 0.41 C. 0.42 D. 0.46 E. 0.50

Solution: $Q_1 \quad Q_2 \quad D_1 \quad D_2 \quad D_3 \quad N \quad P_1 \quad P_2 \quad P_3 \quad P_4$

Notice: 10 coins, $2^{10} = 1024$ cases

the non-quarters add up to 39 cents - so he can't win 50 cents or more without at least one quarter.

the fate of the pennies is completely irrelevant. Might as well just play with the other six coins.

Case 1. Wins both quarters. $P = \frac{1}{4}$

Case 1. Wins just one quarter $P = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$ Then he must add 25 or 30

$\frac{1}{2} \cdot \frac{5}{16} = \frac{5}{32}$ So answer is $\frac{1}{4} + \frac{5}{32} = \frac{13}{32} = 0.40625$

Q_1	Q_2	D_1	D_2	D_3	N	Total	P
		1	1	1	1	35	$\frac{1}{64}$
		1	1	1	0	30	$\frac{1}{64}$
		1	1	0	1	25	$\frac{1}{64}$
		1	0	1	1	25	$\frac{1}{64}$
		0	1	1	1	25	$\frac{1}{64}$
		1	1	0	0	20	
		1	0	1	0	20	
		0	1	1	0	20	

4. Find the sum of all of the distinct positive five digit numbers that can be formed by permuting the digits 1, 3, 5, 7, and 8 (13578, 58371, 83517, 18753, etc.).

A. 1,599,984 B. 6,066,540 C. 6,399,936 D. 15,999,840 E. 31,999,680

Solution: Add them digit by digit. Consider the digit 1. The digit 1 could be the first digit, representing $10^4 = 10,000$ in exactly $4!$ numbers (where we can freely permute the other four numbers). Repeat this idea of 1 in the second place, etc.

$$10\,000 \cdot 24 + 1000 \cdot 24 + 100 \cdot 24 + 10 \cdot 24 + 1 \cdot 24$$

$$(1 + 3 + 5 + 7 + 8) 24 (10\,000 + 1000 + 100 + 10 + 1) = 6399\,936$$

5. There are 150 socks in a bin: 30 blue, 10 pink, 20 green, 40 black, and 50 white. Jerry randomly pulls socks out of the drawer, one at a time, and does not replace them. Let m be the minimum number of socks that he would need to pull out to guarantee that he has at least one matching (same color) pair. Let M be the minimum number of socks that he must pull out to guarantee that he has at least one sock of each color. Find the product of m and M .

A.30 B. 66 C.350 D.705 E.846

Solution: Let's go for m first. The second sock must be of different color than the first. The third different from first and second. And so on, $m = 6$ simply because there are 5 different colors. Even in the worst case scenario, the 6th sock will have a color matching one of the first five. As for M : imagine we get really unlucky with pink and just wouldn't pull one. At the worst case scenario, we need to exhaust the socks until there is only pink left. Then pull one... So, $M = 141$. Then $6(141) = 846$ which is E.

6. Assume that $\sin x + \cos x = \frac{1}{4}$. What is the value of $\sin^3 x + \cos^3 x$?
- A. $\frac{5}{26}$ B. $\frac{11}{32}$ C. $\frac{31}{64}$ D. $\frac{47}{128}$ E. $\frac{59}{256}$

Solution:

$$\begin{aligned} \sin x + \cos x &= \frac{1}{4} && \text{square both sides} \\ \sin^2 x + \cos^2 x + 2 \sin x \cos x &= \frac{1}{16} \\ 1 + 2 \sin x \cos x &= \frac{1}{16} \\ \sin x \cos x &= \frac{1}{2} \left(\frac{1}{16} - 1 \right) \\ \sin x \cos x &= \frac{1}{2} \cdot \frac{-15}{16} = -\frac{15}{32} \end{aligned}$$

$$\begin{aligned} E &= \sin^3 x + \cos^3 x = (\sin x + \cos x) (\sin^2 x - \sin x \cos x + \cos^2 x) = \\ &= \frac{1}{4} (1 - \sin x \cos x) = \frac{1}{4} \left(1 - \left(-\frac{15}{32} \right) \right) = \frac{1}{4} \cdot \frac{47}{32} = \frac{47}{128} \end{aligned}$$

7. Let x, y, z be positive integers such that $x^2 + y^2 + z^7 = 2017$. Find $x + y + z$.
- A. 59 B. 60 C. 61 D. 62 F. 63

Solution: Let's figure out z first. Since $3^7 = 2187$, z can only be 1 or 2.

Case 1. $z = 1$

$$\begin{aligned}x^2 + y^2 + 1 &= 2017 \\x^2 + y^2 &= 2016\end{aligned}$$

$$(x + iy)(x - iy) = 2016 = 2^5 \cdot 3^2 \cdot 7$$

Look at mod 7: both x and y must be divisible by 7

	0	1	2	3	-3	-2	-1	x
0	0	1	4	2	2	4	1	
1	1	2	5	3	3	5	2	
2	4	5	1	-1	-1	1	5	
3	2	3	-1	4	4	-1	3	
-3	2	3	-1	4	4	-1	3	
-2	4	5	1	-1	-1	1	5	
-1	1	2	5	3	3	5	2	

$x = 7A$ $y = 7B$ then $x^2 + y^2$ is divisible by 49 Contradiction.

Case 2. $z = 2$ $2^7 = 128$ and $2017 - 128 = 1889$

$$\begin{aligned}x^2 + y^2 + 128 &= 2017 \\x^2 + y^2 &= 1889\end{aligned}$$

	0	1	2	3	-3	-2	-1	x
0	0	1	4	2	2	4	1	
1	1	2	5	3	3	5	2	
2	4	5	1	-1	-1	1	5	
3	2	3	-1	4	4	-1	3	
-3	2	3	-1	4	4	-1	3	
-2	4	5	1	-1	-1	1	5	
-1	1	2	5	3	3	5	2	

8. How many different ordered 4-tuples of nonnegative integers (a, b, c, d) satisfy the inequality $a + b + c + d \leq 14$?

A. 816 B. 2380 C. 3060 D. 3468 E. 3876

Solution: Let's count! Let's do inductive

$a + b = 0$	0,0							1
$a + b = 1$		0,1	1,0					2
$a + b = 2$		2,0	0,2	1,1				3
$a + b = 3$		0,3	3,0	2,1	1,2			4
$a + b = 4$		0,4	4,0	3,1	1,3	2,2		5
$a + b = 5$		0,5	5,0	4,1	1,4	2,3	3,2	6

So $a + b = n$ has $n + 1$ different solutions

$$\text{and } a + b \leq n \quad 1 + 2 + \dots + n + 1 = \frac{(n+1)(n+2)}{2} = \binom{n+1}{2}$$

solutions.

Now for 3 variables:

$a + b + c = 0$ has just one solution

$a + b + c = 1$ has three solutions, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ 3 solution

Consider now $a + b + c = 2$

$$\text{If } c = 0 \implies a + b = 2 \quad 3$$

$$\text{If } c = 1 \implies a + b = 1 \quad 2$$

$$\text{If } c = 2 \implies a + b = 0 \quad 1 \quad 6 \text{ total}$$

$a + b + c = 3$

$$\text{If } c = 0 \implies a + b = 3 \quad 4$$

$$\text{If } c = 1 \implies a + b = 2 \quad 3$$

$$\text{If } c = 2 \implies a + b = 1 \quad 2$$

$$\text{If } c = 3 \implies a + b = 0 \quad 1 \quad 10 \text{ total}$$

We can prove by induction that $a + b + c = n$ has

$$(n+1) + n + (n-1) + (n-2) + \dots + 3 + 2 + 1 = \frac{(n+2)(n+1)}{2} = \binom{n+2}{2} \text{ many solutions}$$

So, finally, with 4 variables:

$a + b + c + d = 0$ has 1 solution

$a + b + c + d = 1$ has 4 solutions

Consider $a + b + c + d = 2$

$$\text{If } d = 2 \implies a + b + c = 0 \quad 1$$

$$d = 1 \implies a + b + c = 1 \quad 3$$

$$d = 0 \implies a + b + c = 2 \quad 6 \quad \text{Total: } 10$$

Consider $a + b + c + d = 3$

$$\text{If } d = 3 \implies a + b + c = 0 \quad 1$$

$$d = 2 \implies a + b + c = 1 \quad 3$$

$$d = 1 \implies a + b + c = 2 \quad 6$$

$$d = 0 \implies a + b + c = 3 \quad 10 \quad \text{Total: } 20$$

Consider $a + b + c + d = n$

$$\text{If } d = n \implies a + b + c = 0 \quad 1$$

$$d = n - 1 \implies a + b + c = 1 \quad 3$$

$$d = n - 2 \implies a + b + c = 2 \quad 6$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$d = 0 \implies a + b + c = n \quad \binom{n+2}{2}$$

So the total is

$$\begin{aligned}
T &= 1 + 3 + 6 + 10 + \dots + \binom{n+2}{2} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \dots + \binom{n+2}{2} \\
&= \frac{2 \cdot 1}{2} + \frac{3 \cdot 2}{2} + \frac{4 \cdot 3}{2} + \dots + \frac{(n+2)(n+1)}{2} \\
&= \frac{(0+2)(0+1)}{2} + \frac{(1+2)(1+1)}{2} + \frac{(2+2)(2+1)}{2} + \dots + \frac{(n+2)(n+1)}{2} \\
&= \frac{1}{2} ((0^2 + 3 \cdot 0 + 2) + (1^2 + 3 \cdot 1 + 2) + (2^2 + 3 \cdot 2 + 2) + \dots + (n^2 + 3n + 2)) \\
&= \frac{1}{2} ((0^2 + 1^2 + 2^2 + \dots + n^2) + 3(0 + 1 + 2 + \dots + n) + (2 + 2 + \dots + 2)) \\
&= \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} + 3 \left(\frac{(n+1)n}{2} \right) + 2(n+1) \right) \\
&= \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} + \frac{9n(n+1)}{6} + \frac{12(n+1)}{6} \right) \\
&= \frac{1}{12} (n(2n^2 + 3n + 1) + 9n^2 + 9n + 12n + 12) \\
&= \frac{1}{12} (n(2n^2 + 3n + 1) + 9n^2 + 9n + 12n + 12) \\
&= \frac{1}{12} (2n^3 + 3n^2 + n + 9n^2 + 21n + 12) = \frac{1}{12} (2n^3 + 12n^2 + 22n + 12) \\
&= \frac{1}{6} (n^3 + 6n^2 + 11n + 6)
\end{aligned}$$

So $a + b + c + d \leq 14$ is

$$\sum_{n=0}^{14} \frac{1}{6} (n^3 + 6n^2 + 11n + 6)$$

: 3060 which is C

9. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says "I am not a spy.", Y says "X is a knave.", and Z says "Y is a spy." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the spy.	X is the spy.	X is the knight.	X is the knight	X is the knave.
Y is the knight.	Y is the knave.	Y is the knave.	Y is the spy.	Y is the spy.
Z is the knave.	Z is the knight.	Z is the spy.	Z is the knave.	Z is the knight.

Solution: Given the identity of X, there are only 3 cases: X can be a knight, a knave or a spy.

Case 1. Suppose that X is the spy. Then Y lies and so Y must be the knave (spy is taken by X) and so Z must be knight but then Z should tell the truth - but he can't as spy is taken by X. So this case is impossible.

Case 2. Suppose that X is the knight. Then what he says is true, that he is not a spy. Then Y is lying (could be either spy or knave). If Z tells the truth, he must be the spy and thus Y is the knave.

X=knight	Y= knave	Z=spy
I am not a spy	X is a knave	Y is a spy

true	false	false
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So this arrangement works.

Case 3. Suppose that X is a knave. Then his statement must be false and true at the same time. That's impossible.

So: X knight, Y knave, Z spy, which is choice C.

10. Suppose a, b , and c are integers. What is the sum of the reciprocals of the five complex solutions of the equation $x^5 + ax^4 + bx^3 + cx^2 - 12x + 8 = 0$?

A. $-\frac{\sqrt{3}}{2}$ B. -1 C. $\frac{2}{\sqrt{3}}$ D. $\frac{4}{7}$ E. $\frac{3}{2}$

Solution: 1 real, or 3 real or 5 real solutions

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} = \frac{bcde + acde + abde + abce + abcd}{abcde} = \frac{-12}{-8} = \frac{3}{2}$$

11. Let T_n be the number of different ways that a $2 \times n$ grid can be covered by n indistinguishable dominos (1×2 rectangles) with no overlaps or gaps. For example, $T_2 = 2$ is illustrated on the right. Find $T_1 + T_2 + T_3 + T_4 + T_5 + T_6$.

A. 19 B. 32 C. 36 D. 53 E. 64

12. In $\triangle ABC$, $AB = AC = 25$ and $BC = 14$. The perpendicular distances from a

point P in the interior of $\triangle ABC$ to each of the three sides are equal. Find this distance.

A. $\frac{9}{2}$ B. $\frac{19}{4}$ C. 5 D. $\frac{21}{4}$ E. $\frac{11}{2}$

Solution: Consider the area of the triangle. $A = \frac{1}{2}(14)\sqrt{25^2 - 7^2} =$

$$\frac{168}{32} = \frac{21}{4}$$

On the other hand, the same area is

$$\begin{aligned} A &= \frac{1}{2}(64)r = 168 \\ 32r &= 168 \\ r &= \frac{21}{4} \end{aligned}$$

13. A coin has probability p of heads and $1 - p$ of tails. If flipped 3 times, it has probability $\frac{1}{2}$ of producing three flips with the same result (either 3 heads or 3 tails). Find $p(1 - p)$.

A. $1/12$ B. $1/8$ C. $1/6$ D. $1/3$ E. $1/2$

Solution: $p^3 + (1-p)^3 = \frac{1}{2}$

$$\begin{aligned} p^3 + 1 - 3p + 3p^2 - p^3 &= \frac{1}{2} \\ 3p^2 - 3p &= -\frac{1}{2} \\ p^2 - p &= -\frac{1}{6} \\ p - p^2 &= \frac{1}{6} \\ p(1-p) &= \frac{1}{6} \end{aligned}$$

14. Consider a data set that consists of positive integers less than 51. There is exactly one 1 in the data set, and every other integer appears twice as many times as its predecessor appears (so there are exactly two 2s, exactly four 3s, exactly eight 4s, exactly sixteen 5s, etc.) What is the median of this data set?

A. 48 B. 49 C. 49.5 D. 50 E. 51

Solution: $\underbrace{1}_1, \underbrace{2, 2}_2, \underbrace{3, 3, 3, 3}_{2^2}, \underbrace{4, 4, 4, 4, 4, 4, 4, 4}_{2^3}, \dots, \underbrace{49, \dots, 49}_{2^{48}}, \underbrace{50, 50, \dots, 50}_{2^{49}}$

$1 + 2 + 4 + \dots + 2^{48} = 2^{49} - 1$ so the median is 50

$$\underbrace{1, 2, 3, 3, \dots, 49, 49, \dots, 49}_{2^{49}-1} \quad \boxed{50} \quad \underbrace{50, 50, \dots, 50}_{2^{49}-1}$$

15. Suppose $g\left(\frac{1}{x}\right) = \frac{x^2}{2+x}$. Find $g(g(3))$

A. $441/23$ B. $49/9$ C. $9/5$ D. $81/95$ E. $25/207$

Solution: Set $x = \frac{1}{3}$. Then $g(3) = \frac{\left(\frac{1}{3}\right)^2}{\frac{1}{3} + 2} = \frac{\frac{1}{9}}{\frac{7}{3}} = \frac{1}{9} \cdot \frac{3}{7} = \frac{1}{21}$

$g(g(3)) = g\left(\frac{1}{21}\right) = \frac{21^2}{21+2} = \frac{441}{23}$

16. Let N be the greatest 3-digit positive integer that divides all 4-digit numbers with identical digits (those of the form $aaaa$). Let M be the greatest positive integer that cannot be written as $3x + 7y$ for some nonnegative integers x and y . Find $M + N$.

A. 99 B. 110 C. 112 D. 120 E. 121

Solution: N is the greatest 3-digit divisor of $1111 = 11 \cdot 101$ which is 101. Thus $N = 101$. M is 11 and so $N + M = 101 + 11 = 112$

We can get every number beyond that because we can get 11, 12, and 13 (three numbers in a row) and then we can just add multiples of 3 to get greater numbers.

1		7	1·7	13	2·3 + 1·7
2		8		14	2·7
3	1·3	9	3·3	15	5·3
4		10	7+3		
5		11			
6	2·3	12	4·3		

17. Consider $ax^2 + bx + c = 0$, where a is a nonzero rational number and b and c are real numbers. Determine which (if any) of the following statements are true for all possible values of a , b , and c .

I. If both solutions to this equation are rational, then $b^2 - 4ac$ must be equal to the square of a rational number. true

II. If $b^2 - 4ac$ is equal to the square of a rational number, then both solutions to this equation must be rational.

A. Both B. Only I C. Only II D. Neither E. Cannot be determined

Solution: II may not be true if $b^2 - 4ac = 0$. So, solution is B.

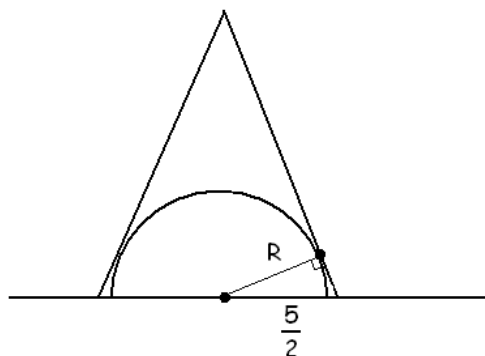
18. Find the area of a semicircle inscribed in an equilateral triangle of side length 5. The diameter of the semicircle lies on one side of the triangle with its center at the midpoint of that side, and the semicircle is tangent to the other two sides.

A. $\frac{20\pi}{8}$ B. 2π C. $\frac{75\pi}{32}$ D. $\frac{25\pi}{4}$ E. $\frac{75\pi}{8}$

Solution:

$$\sin 60^\circ = \frac{R}{\frac{5}{2}} \quad R = \frac{5}{2} \sin 60^\circ = \frac{5}{2} \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$$

$$A = \frac{1}{2}\pi R^2 = \frac{1}{2}\pi \left(\frac{5\sqrt{3}}{4}\right)^2 = \frac{1}{2}\pi \left(\frac{25}{16} \cdot 3\right) = \frac{75\pi}{32}$$



19. Suppose that $f(x) = \frac{x^2 - 16}{ax + b}$ for some real numbers a and b , and that $f(x)$ has an oblique asymptote of $y = 3x + 7$. Find $f(-3)$.
 A. $65/9$ B. $63/16$ C. $35/9$ D. $-7/16$ E. $-35/9$

Solution: oblique asymptote means? : $\frac{x^2 - 16}{ax + b} - (3x + 7) = \frac{A}{x}$

$$\begin{aligned} E &= \frac{x^2 - 16}{ax + b} - (3x + 7) = \frac{x^2 - 16 - (3x + 7)(ax + b)}{ax + b} = \frac{x^2 - 16 - 3ax^2 - 3bx - 7ax - 7b}{ax + b} \\ &= \frac{(1 - 3a)x^2 - 16 - 3bx - 7ax - 7b}{ax + b} \quad 1 - 3a = 0 \quad a = \frac{1}{3} \\ &= \frac{-3bx - 7 \cdot \frac{1}{3}x - 7b - 16}{\frac{1}{3}x + b} = \frac{\left(-3b - \frac{7}{3}\right)x - 7b - 16}{\frac{1}{3}x + b} \quad 3b + \frac{7}{3} = 0 \quad b = -\frac{7}{9} \end{aligned}$$

$$\text{So } f(x) = \frac{x^2 - 16}{ax + b} = \frac{x^2 - 16}{\frac{1}{3}x - \frac{7}{9}} = \frac{9x^2 - 144}{3x - 7} \text{ and so } f(-3) = \frac{9(-3)^2 - 144}{3(-3) - 7} =$$

$$\frac{81 - 144}{-16} = \frac{63}{16}$$

Previously:

If we divide, the quotient is $3x + 7$ and the remainder is $r \in \mathbb{R}$.

$$\begin{aligned} (ax + b)(3x + 7) + r &= x^2 - 16 \\ 3ax^2 + x(7a + 3b) + \frac{7}{3}b + r &= x^2 - 16 \implies 3a = 1 \implies a = \frac{1}{3} \\ x^2 + x\left(\frac{7}{3} + 3b\right) + \frac{7}{3}b + r &= x^2 - 16 \implies \frac{7}{3} + 3b = 0 \implies b = -\frac{7}{9} \end{aligned}$$

$$\text{Thus } f(x) = \frac{x^2 - 16}{ax + b} = \frac{x^2 - 16}{\frac{1}{3}x - \frac{7}{9}} = \frac{9x^2 - 144}{3x - 7} \text{ and so } f(-3) = \frac{9x^2 - 144}{3x - 7} = \frac{63}{16}$$

20. How many positive integers less than or equal to 1000 have an equal number of even and odd factors? For example, 10 would be counted since it has two odd (1 and 5) and two even (2 and 10) factors.

A. 100 B. 125 C. 200 D. 250 E. 500

Solution: Let q be an odd number. Then $n = 2q$ would work.

All the odd numbers from 1 to 500: 250 of them. Their doubles are the numbers.

$$\sqrt{x+1} + \sqrt{x} = 15 \quad \text{or} \quad \sqrt{x+1} + \sqrt{15} = 15$$

$$\sqrt{x+1} = 15 - \sqrt{x}$$

$$x+1 = 225 - 30\sqrt{x} + x$$

$$30\sqrt{x} = 224$$

$$x = \left(\frac{224}{30}\right)^2 = \left(\frac{112}{15}\right)^2 = \frac{12544}{225}$$

$$\text{or } \sqrt{x+1} + \sqrt{15} = 15$$

$$\sqrt{x+1} = 15 - \sqrt{15}$$

$$x+1 = (15 - \sqrt{15})^2$$

$$\begin{aligned} x &= (15 - \sqrt{15})^2 - 1 = 15(\sqrt{15} - 1)^2 - 1 = 15(16 - 2\sqrt{15}) - 1 = \\ &= 240 - 30\sqrt{15} - 1 = 239 - 30\sqrt{15} \end{aligned}$$