

Problem Set 1

1. A father has three sons. The age of the father is one more than three times his oldest son's age. The age of the oldest son equals to the sum of his two younger brothers' age. The smallest son is a year younger than the middle one. Seven years from now, the age of the father will equal the sum of his sons' age. How old is the father, how old are his sons?
2. The vertices of a triangle are $A(\lg x, \lg y)$, $B(\lg 4x, \lg y)$, and $C(\lg x, \lg 8y)$. Find the area of the triangle. Find the value of x and y so that the circle written around the triangle is centered around the point $O(2, 2)$.
3. A right trapezoid has parallel sides AB and CD ($AB > CD$), a perpendicular side AD . The diagonal AC equals to the side AB and the side AD equals to the side DC . A line through C , perpendicular to side BC intersects side AD in the point F . Find the area of the trapezoid if we know that $DF = 1$.
4. Prove the following inequality:

$$\sqrt{1993} + \sqrt{1994} + \sqrt{1996} + \sqrt{1997} < 4\sqrt{1995}$$

5. Find the product of the different prime numbers p_1, p_2, p_3 , and p_4 if

$$p_1^2 - (p_2 + p_3 + p_4)^2 = 136$$

6. Consider the function $f(x) = 3x^2 + 4x(\cos \alpha - \sin \alpha) - 3 \sin 2\alpha$. Find the value of α for which the minimum of $f(x)$ is smallest.
7. Solve the following equation:

$$x^{\log_3(x-1)} + 2 \cdot (x-1)^{\log_3 x} = 3x^2$$

8. A prism's sides are positive integers, greater than one. The volume of the prism is 72. If we knew the sum of the surface area's digits, we still couldn't determine the sides. But then we are told that the smallest side is positioned vertically. Now we can find the sides. Can you?