

1. The **Fibonacci sequence**, F_n is 1, 1, 2, 3, 5, 8, 13, 21, ... Every element is obtained by adding the previous two elements. The recursive definition is

$$\begin{aligned}F_1 &= 1 \\F_2 &= 1 \\F_{n+2} &= F_n + F_{n+1} \quad \text{for all natural number } n, \text{ with } n \geq 1\end{aligned}$$

Consider now another sequence, q_n that is formed by taking the ratios of consecutive elements of the Fibonacci sequence.

$$q_n = \frac{F_n}{F_{n+1}} \quad \text{for all natural number } n.$$

Compute the decimal presentation of the first few elements of q_n . What behavior do you notice?

2. Prove that consecutive elements of the Fibonacci sequence are relatively prime.
3. Leo the 58th is a king in a country. Amazed by his own greatness, he announces that the use of all currency is illegal, except for the use of \$ 5 bills and \$ 8 bills.
- (a) We see a product in a store that costs \$ 7. Can we purchase it with the legal currencies?
 - (b) Is there a price that would be impossible to pay?
 - (c) What would be the case if his son, Leo the 59th started to rule?
 - (d) Ask further questions!

4. Definition: (All letters denote positive integers) A number n is **irreducible** if for all a and b ,

$$n = ab \implies a = 1 \text{ or } b = 1$$

Definition: A number n is **prime** if for all a and b

$$n|ab \implies n|a \text{ or } n|b$$

- (a) Prove that if a number is prime, then it is irreducible.
- (b) Prove that if a number is irreducible, then it is a prime.