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In memory of László Párkányi, teacher

translated from: KoMaL, December 2000, pages 523-524

Pdf file: KoMaL, December 2000, pages 523-524

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. In one year, a factory increased its production by 26.5%. This increase came from two sources: the number of workers increased, and the production per worker increased. The percent increase of the production per worker was 1.5 times the percent increase of the number of workers. By what percent did each of them increase?
2. In the triangle ABC , $AB = 10$ units, $A(0, 6)$, the point B is on the x -axis, and the centroid of the triangle is $S(4, 6)$. Find the area of the triangle.
3. For which real numbers x are the values of the following expressions positive?
a) $\frac{x^4 - 16}{x^2 - 4}$ b) $|x + 2| - |x - 2| - x$ c) $1 + \log_2 \sin 2x$
4. The sum of the first n terms of the sequence (a_n) is $S_n = 2n^2 + 3n$ for every positive integer n . The n th term of the sequence (b_n) is $b_n = 4n - 1$.
a) Prove that (a_n) and (b_n) are arithmetic sequences. Write the n th term of (a_n) as a function of n .
b) Express the sum of the first n terms of (b_n) in terms of n .
c) Find the sum of the first n terms of the sequence $c_n = a_n + b_n$.
5. Solve the following equation over the real numbers.

$$\cos x - \sin x = \frac{\cos 2x}{1 + \sin 2x}$$

6. One angle of a triangle is 75° , and the altitude from the vertex of this angle divides the opposite side into parts of lengths $\sqrt{6}$ and $\sqrt{2}$. Find the other two angles of the triangle.
7. Find the values of the real parameter p for which the roots of the equation $x^2 + 4px + 2p^2 + 3p - 1 = 0$ satisfy the following conditions.
a) The two real roots are equal. b) The two real roots are reciprocals of each other.
c) One real root is twice the other real root. d) The product of the real roots is as small as possible.
8. A pan of pastry is cut into pieces with straight cuts parallel to the sides of the pan, each running from side to side. The pieces along the edges are called "bad", and the others "good". How many cuts should we make so that the pastry can be divided into servings of two "good" pieces and one "bad" piece each? How many servings can be made this way?