Problem Set III - 2000 (February 2000)

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In memory of János Radó, principal (my friend)
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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following system of equations over the pairs of real numbers.

$$x^2 - y^2 = 2(x - y), \qquad x^2 + y^2 = 5(x + y)$$

2. The product of the first and fifth terms of a geometric sequence is 144, and the difference of the second and fourth terms is 18. Find the first term and the ratio of the sequence.

3. Find the equation of the circle that is tangent to the y-axis at the origin and is also tangent to the line $x + y = 1$.

4. Find the values of the real parameter m for which the roots of the equation

$$2x^2 + (2 - 2m)x + \left(m^2 - 4m + 1\right) = 0$$

are real, and for these values of m find the largest and the smallest values of $x_1 \cdot x_2$, where x_1 and x_2 are the roots of the equation.

5. Let $x > 0$, $y > 0$, and $x + y = 4$. Prove that

$$\left(3 + \frac{1}{x}\right)\left(3 + \frac{1}{y}\right) \geq \left(\frac{7}{2}\right)^2$$

6. Two sides of a triangle are 12 and 24 units long, and the bisector of the angle between them is 8 units long. Find the angle between the two sides and the third side.

7. A point P of the line of the side AB of the square $ABCD$ is 5 units from the vertex C and $\sqrt{17}$ units from the vertex D. What is the side of the square? Where is the point P?

8. Solve the following equation over the pairs of real numbers.

$$4\cos^2 x - 4\sin x \cdot \sin y - 5 = 0$$

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