

author: Imre Rábai

*In memory of Professor Tibor Gallai*

We recommend that students work through the problems of these practice sets with the help of a teacher, and that the problems be discussed in class or in a math club after they are solved.

translated from: KoMaL, October 2000, page 400

Pdf file: KoMaL, October 2000, page 400

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The sum of the first, third, and fifth terms of a geometric sequence is 126, and the sum of the second and fourth terms is 30 times the ratio. Find the first term and the ratio of the sequence.
2. Prove that if  $a > 0$ ,  $b > 0$ ,  $c > 0$ , and  $a + b + c = 3$ , then

$$\sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1} < 9$$

3. In the rectangle  $ABCD$ ,  $AB = 20$  and  $BC = 10$  units. There is a point  $P$  inside the rectangle from which both segments  $AB$  and  $AD$  are seen at a  $90^\circ$  angle. Find the distances of  $P$  from the vertices of the rectangle.
4. Solve the following equations over the real numbers.
  - a)  $\log_3 (x-1)^2 + \log_3 (x+2)^2 = 2 \log_3 2$
  - b)  $\log_3 (x-1)^2 + 2 \log_3 (x+2) = 2 \log_3 2$
  - c)  $2 \log_3 (x-1) + \log_3 (x+2)^2 = 2 \log_3 2$
  - d)  $2 \log_3 (x-1) + 2 \log_3 (x+2) = \log_3 4$

5. The sides of a triangle are  $a$ ,  $b$ , and  $c$ , and its area is

$$T = \frac{\sqrt{3}}{4} (c^2 - a^2 - b^2)$$

Express the radius of the circle circumscribed around the triangle in terms of  $a$  and  $b$ .

6. The roots of the equation  $x^2 - a^2x - 2ab = 0$  are 2 greater than the roots of the equation  $x^2 + 3ax - \frac{4}{3}b = 0$ . Find the values of  $a$  and  $b$ , and the roots of the equations.
7. Find the pairs of real numbers  $(x, y)$  with  $0 \leq x \leq 2\pi$  and  $0 \leq y \leq 2\pi$  that satisfy the following system of equations.
$$2 \cos^3 x + \sin y = 0, \quad 2 \sin^3 x + \cos y = 0$$
8. Two vertices of the triangle  $ABC$  are  $A(7, 9)$  and  $B(-1, 3)$ ,  $BC = 4\sqrt{5}$  units, and the area of the triangle is 40 square units. Find the coordinates of the vertex  $C$ .