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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The diagonals of a parallelogram are 13 and 20 units long, and one of its sides is 10.5 units long. Find the area and the perimeter of the parallelogram.
2. Solve the following equation over the real numbers.

$$\log(x+9) + \log 4 = 2 - \frac{1}{2} \log(x-1)^2$$

(log denotes the base 10 logarithm.)

3. The vertices of a triangle are $A(-2, 0)$, $B(2, 0)$, and $C(0, 5)$. Find the set of points P in the plane for which

$$PA^2 + PB^2 + PC^2 = 30$$

4. For the geometric sequence (a_n) , $7(a_2 + a_3) + 2(a_1 + a_4) = 0$ and $S_5 = 22$. Find the first term and the ratio of the sequence.
5. Find the exact value of $\sin x$, if $\tan x + \cot x = -4$.
6. Consider the function $x \mapsto (p-1)x^2 + 2(p+2)x + p-2$, where x is real and p is a real parameter. Find the value of p for which the largest value of the function is -6 . Where does the function take this largest value?
7. One root of the equation $x^2 + px + q = 0$ is 4, and twice the other root is equal to the discriminant of the equation. Find p and q .
8. A parabola whose axis is parallel to the y -axis is tangent to the x -axis and passes through the point $A(3, 2)$. A normal vector of the tangent line to the parabola at A is $\mathbf{n}(4, 1)$. Find the equation of the parabola.