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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The midpoints of the sides of a convex quadrilateral are the vertices of a rhombus. The side of the rhombus is 6 units, and one of its angles is 30° . Find the area of the quadrilateral. (How many such quadrilaterals are there?)

2. Solve the following inequality over the real numbers.

$$\log_x (-2x^2 - 2x + 4) \geq \log_x (4 - 3x)$$

3. The sides a, b, c_1 of one triangle satisfy

$$\frac{(a+b)^2 - c_1^2}{ab} = 2 - \sqrt{2}$$

and the sides a, b, c_2 of another triangle satisfy

$$\frac{c_2^2 - (a-b)^2}{ab} = 2 + \sqrt{2}$$

Is it true that the two triangles have the same area?

4. Solve the following equation over the real numbers.

$$\sqrt{x+4+2\sqrt{x+3}} - \sqrt{2x+3-4\sqrt{2x-1}} = 3$$

5. Consider the function

$$x \mapsto f(x) = 3\sqrt{3}\sin^2 x + 6\sin x \cos x - 3\sqrt{3}\cos^2 x, \quad x \in \mathbb{R}$$

Find the largest and the smallest value of the function, and the values of x where these extreme values are taken.

6. Four numbers are four consecutive terms of an arithmetic sequence. If we add 1 to the first number, subtract 6 from the second, subtract 4 from the third, and add 4 to the fourth, then we get four consecutive terms of a geometric sequence. What are the four numbers?
7. The circle k is tangent to the x -axis, and is tangent to the line $e: 3x - 4y + 45 = 0$ at the point of e whose y -coordinate is $y_0 = 9$. Find the equation of the circle k .
8. For which value of the real parameter a is the sum of the squares of the real roots of the equation $x^2 - 2x + (a-1)^2 = 0$ smallest, and for which value is it largest? What are the smallest and largest values?