Problem Set III - 2001 (February 2001)

author: Imre Rábai
In memory of Ottó Steinfeld, mathematician
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Pdf file: KoMaL, February 2001, page 88

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. In the triangle ABC, $AC = 5.8$ and $BC = 5$ units. The altitude to the side AB is $CT = 4$ units. Find the area of the triangle.

2. Solve the following system of equations over the pairs of real numbers.

$$x^2 = 2y + 8, \qquad y^2 = 2x + 8$$

3. In the triangle ABC the bisector of the interior angle at C meets the side AB at D. Find the angle of the triangle at C and the length of the side AB, if $AC = 5$, $BC = 12$, and $CD = \dfrac{60\sqrt{2}}{17}$ units.

4. Solve the following inequalities over the real numbers.
   a) $\sqrt{4x - x^2} > 6 - 2x$ b) $\log_{\frac{3}{4}} \log_{18}\left(x^2 - 63\right) \geq 0$

5. In the rectangle $ABCD$, $BC = 3AB$. The midpoints of the shorter sides are $E\left(\dfrac{7}{2}, -\dfrac{3}{2}\right)$ and $F\left(-\dfrac{11}{2}, \dfrac{3}{2}\right)$. Find the coordinates of the vertices.

6. Find the positive integers a, b, and c for which

$$ab = a + 2b + 33$$

   and the segments of lengths a, b, and c form a triangle whose three sides are all divisible by 3.

7. What are the possible values of $2p - 2q$, if p and q are real numbers such that the roots of the equation $x^2 - px + q = 0$ are real, and the roots x_1 and x_2 satisfy $x_1^2 + x_2^2 = 4$?

8. Solve the following equation, where p is a real parameter.

$$\frac{x - 2p}{x + 2p} = \frac{x - p}{x - 2p} - \frac{8p^2 - 3x}{x^2 - 4p^2}$$

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