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In memory of Zsigmond Kokits, mathematician, associate professor

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

a) $\frac{x-3}{x-2} + \frac{10}{x+2} = \frac{44}{4-x^2}$

b) $\frac{x-13}{x-2} + \frac{10}{x+2} = \frac{44}{4-x^2}$

c) $\frac{x-4}{x-2} + \frac{10}{x+2} = \frac{44}{4-x^2}$

d) $\frac{x-8}{x-2} + \frac{10}{x+2} = \frac{44}{4-x^2}$

2. One side of a rectangle is four times as long as the other side. Find the ratio of the area of the rectangle to the area of the quadrilateral bounded by the angle bisectors of the rectangle, and the ratio of the diagonal of the rectangle to a side of this quadrilateral.

3. Find the smallest and the largest value of the expression $f(x) = \frac{3x^2 + 11x - 34}{x^2 + 4x - 12}$ on the interval $[-4, -1]$.

4. Solve the following equation over the real numbers.

$$3 \tan \left(x + \frac{3\pi}{2} \right) = \tan 2x$$

5. Prove that for every positive integer n

$$S_n = 4 \cdot 6 + 5 \cdot 13 + \dots + (n+3)(7n-1) = \frac{n}{6} (14n^2 + 81n + 49)$$

6. a) The discriminant of the equation $x^2 + px + q = 0$ (p, q real) is $D \geq 0$, and the roots of the equation are x_1 and x_2 . Prove that $(x_1 - x_2)^2 = D$.

b) The real roots of the equation $x^2 - 2ax + 2a^2 - 3a = 0$ (a real) are x_1 and x_2 . For which value of the parameter a is $(x_1 - x_2)^2$ largest? What is this largest value?

7. The vertices of the triangle ABC are $A(0, 16)$, $B(0, 0)$, and $C(3, 0)$. Find the equation of the line that passes through the point $P_0(-6, 0)$ and divides the triangle into two parts of equal area.

8. a) Prove that if $xy + yz + zx = 1$ (x, y, z real), then $x^2 + y^2 + z^2 \geq 1$.

b) Consider the rectangular boxes whose space diagonal is 2 units long. Which of these boxes has the largest surface area, and what is this largest surface area?