

author: Imre Rábai

translated from: KoMaL, October 2001, page 395

Pdf file: KoMaL, October 2001, page 395

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Find the sides of a right triangle, if its area is $T = 43.2$ square units and the radius of its inscribed circle is $\varrho = 2.4$ units.

2. Prove that the equation

$$2x^2 - 2(b+c)x + (ab+bc+ca-a^2) = 0$$

has a real solution for all real numbers a, b, c . When are the two solutions of the equation equal, and what is the solution then?

3. The parallel sides of an isosceles trapezoid are a and b ($a > b$), and the diagonals make an angle α ($0^\circ < \alpha < 90^\circ$) with the parallel sides. Express the radius of the circle circumscribed around the trapezoid in terms of a, b , and α .
4. The first term of a geometric sequence is 3 more than the ratio of the sequence, and there is a positive integer n for which the sum of the first n terms is 35, and the product of the first term and the n th term is 100. Find the first term, the ratio, and n .
5. Find the equation of the line that passes through the point $P(5, 3)$ and meets the lines $3x + 2y = 16$ and $3x + 2y = 11$ at points whose x -coordinates differ by 1.
6. Solve the following equation over the pairs of real numbers.

$$x^2 - 4x \sin(xy) + 4 = 0$$

7. The lateral edges of a pyramid with a rectangular base are all equal, with length c , and the sides of the rectangle are a and b . At what distance from the apex should we place a plane parallel to the base so that the plane divides the pyramid into two parts of equal volume? Express the area of the cross section in terms of a, b , and c .
8. What are the possible values of $q - p$, if p and q are real numbers such that the roots of the equation $x^2 - px - q = 0$ are real, and the roots satisfy $x_1^2 + x_2^2 = \frac{9}{2}$?