

author: Imre Rábai

In memory of Géza Fodor, mathematician, university professor

translated from: KoMaL, April 2002, page 199

Pdf file: KoMaL, April 2002, page 199

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The sides of a rectangle are 15.6 and 18 units long. We inscribe a rectangle in it, one side of which is three times as long as the other side, and which has exactly one vertex on each side of the given rectangle. Find the sides of the inscribed rectangle.
2. a) Prove that in every triangle $\cot \alpha \sin \gamma + \cos \gamma = \frac{b}{a}$, where α is the angle opposite a and γ is the angle opposite c .
b) In a triangle $b = 2a$ and $\gamma = 60^\circ$. Find the other two angles of the triangle.
3. For the geometric sequence (a_n) , $a_1 + a_3 + a_5 = 133$ and $\frac{1}{a_1} + \frac{1}{a_3} + \frac{1}{a_5} = \frac{133}{1296}$. Write down the first five terms of the sequence.
4. We deposit 40 000 forints in a bank for 3 years at compound interest. In the first year the bank pays 10% interest; in the second year it increases the interest rate by $p\%$, and in the third year it increases it by another $p\%$ (that is, by p percentage points each time). This way, at the end of the third year, the bank pays 2939.20 forints more than it would have paid at 10% interest for all three years. Find the value of p .
5. The vertex A of the quadrilateral $ABCD$ in the coordinate plane is at the origin, B is on the y -axis, C is in the first quadrant, and D is on the positive half of the x -axis. The equation of the line of the side BC is $2x + y = 12$. The side CD is perpendicular to the side BC . The area of the quadrilateral is 31 square units. Find the coordinates of the vertices B , C , and D .
6. The radii of the bottom and top circles of a frustum of a cone are R and r ($R > r$). A plane parallel to the bases divides the frustum into two parts so that the frustum next to the larger base has one third of the volume of the original frustum. Express the radius of the cross section in terms of R and r .
7. Prove that the equation $(\sin x - \sqrt{3} \cos x) \cdot \cos 4x = 2$ has no solution over the real numbers.
8. Consider the equation $4^x + 2(n+1) \cdot 2^x + n^2 = 8$, where n is an integer parameter.
 - a) For which n does the equation have two different real solutions?
 - b) For which n does the equation have exactly one solution?Find the solutions too.