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In honor of György Ágoston, university professor (pedagogy)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

- The terms of the sequence (a_n) are the positive integers that give remainder 5 when divided by 8, and the terms of the sequence (b_n) are the positive integers that give remainder 1 when divided by 6 (both in increasing order). Let S_n and T_n be the sums of the first n terms of (a_n) and (b_n) .
 - For which n is $\frac{S_n}{T_n} = \frac{25}{16}$?
 - What is the smallest n for which $S_n - T_n > 1000$?
- The position vectors of the points A and B are $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point of trisection of the segment OA closer to O is D , the point of trisection of the segment AB closer to A is E , and the lines BD and OE meet at P . Express the vectors $\overrightarrow{OP}$ and $\overrightarrow{PD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
- Solve the following equations over the real numbers.
 - $\sqrt{1 - 4x + 4x^2} + 2x - 1 = 0$
 - $x^{\log_3 x} = \sqrt[9]{81}$
 - $\sin 2x + 7 \cos 2x = 1$
- Consider the regular square pyramid all of whose edges are equal, and the radius of whose inscribed sphere is $\varrho = 1$ unit. Find the length of the edges.
- In the triangle ABC the angle at A is twice the angle at B , $6AB = 5BC$, and the area of the triangle is 62.5 times the sine of the angle at A . Find the sides of the triangle.
- Find the equation of the circle that passes through the point $P(-1, -1)$ and is tangent to the lines $x + 2y - 6 = 0$ and $2x - y + 3 = 0$.
- Solve the following equations over the real numbers, where a is a real parameter.
 - $(a + 2) \cdot y + \frac{a}{y - 2} = 4a + 6$
 - $(a + 2) \cdot 3^x + \frac{a}{3^x - 2} = 4a + 6$
- Which pairs of real numbers satisfy the following equation?

$$x\sqrt{\log_x y} = y\sqrt{\log_y x}$$