

author: Imre Rábai

In memory of István Pásztor, mathematician, associate professor

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance. The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The population of a town is now 86 000. The population grows by 5% every year. What was the population 3 years ago? By what percent did the population grow in these three years?
2. Solve the following system of equations over the pairs of real numbers.

$$x^{\log_3 y} + 2 \cdot y^{\log_3 x} = 27, \quad \log_3 y - \log_3 x = 1$$

3. On each side of a square with side $2\sqrt{3}$ units, we construct an isosceles triangle inside the square, whose angle between the legs is 120° . Find the area of the quadrilateral whose vertices are the vertices of these triangles inside the square.
4. The hypotenuse of the right triangle ABC is $AB = 2\sqrt{3}$ units. The midpoint of the hypotenuse is C_1 , and the midpoint of the leg BC is A_1 . The median CC_1 is perpendicular to the median AA_1 . Find the lengths of the legs.
5. The areas of two faces of a rectangular box are 12 and 36 square units, and its space diagonal is 13 units long. Find the surface area and the volume of the box.
6. For which natural numbers n are exactly two of the following statements true?
a) $120n - 4n^2 - 899 > 0$ b) $n + 1$ is divisible by 7 c) $n^2 - 1$ is divisible by 7
7. Find the values of the parameter α in the interval $[0, 2\pi]$ for which the roots of the equation $(2 \sin \alpha + 1)x^2 - 4x + 4 \sin \alpha - 2 = 0$ have opposite signs.
8. How many lines pass through the point $B(4, 3)$ that meet the x -axis at a point with an integer x -coordinate, and meet the positive half of the y -axis at a point whose y -coordinate is a prime? Find the equations of these lines.