

author: Gyula Orosz

translated from: KoMaL, January 2002, pages 30-31

Pdf file: KoMaL, January 2002, pages 30-31

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The real numbers a, b, c, x, y, z satisfy $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. What is the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$?
2. Solve the equation $4 \cdot \log_x 2 = 7 + 2 \cdot \log_2 x$ over the real numbers.
3. One root of the equation $x^3 + x^2 - 14x + m = 0$ is 3. Find the value of the real parameter m and the other two roots.
4. In the triangle ABC the altitude from B meets the line AC at D , and the altitude from C meets the line AB at E . How long is the segment DE , if $\angle CAB = 70^\circ$, $AB = 12$ cm, and $AC = 10$ cm?
5. Solve the inequality $\cos 2x - 3 \cos x + 2 \leq 0$ over the real numbers.
6. At most how many different positive primes can be given so that the sum of any three of them is also a prime?
7. Which of the right circular cylinders inscribed in a sphere of radius R has the largest volume? What is the largest possible volume?
8. Consider the parabola $y = x^2$ and the point $A(9, 8)$. At which points P of the parabola is the tangent line perpendicular to the line AP ?