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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The trapezoid $ABCD$ has right angles at the vertices A and D , the diagonals AC and BD are perpendicular, and $4AB = 9CD$. Find the ratio of the lengths of the diagonals.
2. Solve the following equation, where p is a nonzero real parameter.

$$\frac{x-14}{2} + \frac{x-12}{4} + \frac{x-10}{6} = \frac{x-8}{8} + \frac{x-6}{10} + \frac{x-4}{12} + \frac{x-16}{p}$$

3. Find the equation of the circle that passes through the points $A(0, 9)$ and $B(7, 2)$ and is tangent to the x -axis.
4. The base of a pyramid is a trapezoid with three sides 6 units long and the fourth side 12 units long. The lateral edges of the pyramid make equal angles with the plane of the base. What is this angle, if the volume of the pyramid is 162 cubic units?
5. The lines that trisect an angle of a triangle divide the opposite side into segments of lengths 5, 6, and 9 units. Find the exact value of the cosine of this angle.
6. Find the smallest positive solution of the following equation.

$$2 \sin \frac{2002^{100} \cdot \pi}{-3} \cdot 5^{\frac{1}{\log_3 25}} = \left| \sqrt{x^2 - 2x + 1} - 4 \right|$$

7. For which pairs of positive primes p and q does the equation

$$(p^2 - pq - 2q^2)x^2 - 3q^2x - (2p + 0.5) = 0$$

have exactly one real root?

8. For which real numbers is the following expression defined?

$$\sqrt{\frac{2^{-x+1} - 0.5}{\log_x(x^2 + x)}}$$