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In memory of György Geréb, psychologist, college professor

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance. The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. One of the parallel sides of an isosceles trapezoid that has an inscribed circle is 10 units long, and the radius of the inscribed circle is $\varrho = 5\sqrt{3}$ units. What percent of the area of the trapezoid is the area of the inscribed circle?
2. The distance between two towns is 560 km. One of two cars covers this distance in 1 hour less than the other car, because its speed is 10 km/h greater. Find the speeds of the cars.
3. The points $P(b, -2)$ and $Q(16b, 6)$ are on the graph of the logarithm function $f : x \mapsto \log_a x, x > 0 (a > 0, a \neq 1)$. Find a and b , and write down the equation of the line PQ .
4. For which real numbers x is the following expression defined, and what is its range?

$$\frac{\sin 2x + 2 \sin x}{\sin 2x - 2 \sin x} \cdot \tan^2 \frac{x}{2}$$

5. Find the equation of the circle of radius $\sqrt{8}$ that passes through the point $P(0, 2)$ and is externally tangent to the circle $x^2 + y^2 + 2x + 6y + 8 = 0$.
6. Solve the following inequality over the real numbers.

$$2\sqrt{\log_3 x} + \sqrt{\log_x 3} \geq 3$$

7. A convex hexagon is inscribed in a circle of radius $\sqrt{13}$ units. Three of its sides are $2a$ and the other three sides are $5a$ units long. Find the area of the hexagon.
8. The first term of a sequence is $a_1 = 1$, and for $n \geq 1$

$$a_{n+1} = \left(1 - \frac{1}{(n+2)^2}\right) \cdot a_n$$

Write the n th term of the sequence and the product of its first n terms in closed form.