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*In honor of Gergő (Grünwald) Gézáné, mathematician, university assistant professor*

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance. The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The sides of the triangle  $ABC$  are  $AB = 20$ ,  $BC = 13$ , and  $CA = 11$  units. Find the area of the triangle, and the radii of its inscribed and circumscribed circles.
2. Solve the following equation over the real numbers.

$$\left(x^2 + x + 1 + \frac{2}{x-1}\right) \left(x^2 - x + 1 - \frac{2}{x+1}\right) = x^4 + x^2 + 1$$

3. The first term of the arithmetic sequence  $(b_n)$  is  $b_1 = 3$  and its difference is  $d_1 = 4$ ; the first term of the arithmetic sequence  $(c_k)$  is  $c_1 = 2$  and its difference is  $d_2 = 7$ . The common terms of the two sequences form the sequence  $(a_m)$ . Express the sum of the first  $m$  terms of  $(a_m)$  in terms of  $m$ .
4. Two sides of a triangle are  $b$  and  $c$  units long, and the area of the triangle is  $T = a^2 - (b - c)^2$ . Express the side  $a$  in terms of  $b$  and  $c$ .
5. Solve the following system of equations over the pairs of real numbers.

$$3^{1+2\log_3(y-x)} = 48, \quad 2\log_5(2y - x - 12) = \log_5(y - x) + \log_5(y + x)$$

6. Solve the following equation over the real numbers.

$$2\sin 2x = 2\sqrt{6}\sin\left(x - \frac{\pi}{4}\right) + 3$$

7. The line  $x + y = 15$  meets the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ . Find the equation of the line that is perpendicular to the line  $3x + y = 2002$ , meets the segment  $OB$  at  $C$  and the segment  $AB$  at  $D$ , and for which the area of the quadrilateral  $OADC$  is 58.5 square units, where  $O$  is the origin.
8. Prove that if  $x^2 + y^2 = 18$  for a pair of real numbers  $(x, y)$ , then  $|x + y| \leq 6$ . For which pairs does equality hold?