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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Find the four-digit number $\overline{abcd}$ for which

$$\overline{abcd} + \overline{abc} + \overline{ab} + \overline{a} = 2003$$

2. The diagonals of a trapezoid are perpendicular, one of them is 5 units long, and the height of the trapezoid is 4 units. What is its area?
3. Solve the following equation over the pairs of real numbers.

$$|\sin x + \cos x| = \sqrt{2} (y^2 + 2y + 2)$$

4. The sides of a triangle enclosing the angle α are $\sin \alpha$ and $\cos \alpha$ units long, and the third side is $\sqrt{1 - \sin \alpha \cdot \cos \alpha}$. Is it true that the area of the triangle is a rational number?
5. Find the values of the real parameter a for which the domain of the following expression is the empty set.

$$\sqrt{(a^2 - 2a - 3) \cdot x^2 + (2a - 3) \cdot x + 1}$$

6. In an increasing geometric sequence, the sum of the first and the n th term is 66, the product of the second and the $(n - 1)$ th term is 128, and the sum of the first n terms is 126. Write down the first n terms of the sequence.
7. Solve the following equation over the real numbers, where a and b are real numbers with $a \neq 0$, $b \neq 0$, and $|a| \neq |b|$.

$$\frac{a + bx}{a - bx} - \frac{a - bx}{a + bx} = \frac{4ab}{a^2 - b^2}$$

8. Find the set of real numbers that satisfy both of the following inequalities.

$$(1) \quad x^{\log_{36}(x+6) - \log_6 x} > 1 \qquad (2) \quad \sin \left(\cos x + \frac{1}{2} \right) \leq 0$$