Problem Set - 2003 (February 2003)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equation over the real numbers.

$$\log_{x-1}(x-2)\cdot\log_{x-2}(x-3)\cdot\log_{x-3}(x-1)=1$$

2. The edges of a rectangular box (of three different lengths) are in the ratio $1:2:3$. We decreased every edge by 4 cm, and so the surface area decreased by 1056 cm^2. By how much did the volume of the box decrease?

3. Solve the following system of equations over the pairs of integers.

$$\frac{20}{x+y+2}-\frac{6}{3x+y-5}=1,\qquad \frac{12}{x+y}+\frac{21}{3x+y}=7$$

4. Show that the sides of a triangle satisfy $a^2b-bc^2+c^3=a^2c+b^2c-b^3$ if and only if the triangle is a right triangle or an isosceles triangle.

5. The roots of the equation $x^2-6x+4=0$ are the lengths of two sides of a rectangle, in centimeters. The axial cross sections of two noncongruent cylinders are congruent to this rectangle. Find the sum of the surface areas and the sum of the volumes of the two cylinders.

6. Four siblings inherited a plot of land in the shape of a convex quadrilateral. They divided it into four quadrilaterals by joining the midpoints of opposite sides. The first three siblings received plots of 360, 720, and 900 m^2. How large a plot did the fourth sibling receive?

7. Prove that if $\alpha>0$, $\beta>0$, $\gamma>0$, and $\alpha+\beta+\gamma=\dfrac{\pi}{2}$, then

$$\tan\alpha\tan\beta+\tan\beta\tan\gamma+\tan\gamma\tan\alpha=1$$

8. Solve the following equation over the real numbers.

$$x^2+\frac{9x^2}{(x+3)^2}=40$$

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