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translated from: KoMaL, March 2003, page 142

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Simplify the following expression.

$$\left[\frac{1 + \sqrt[3]{x} + \sqrt[3]{x^2}}{1 - x} - \left(x^{-\frac{2}{3}} - x^{-\frac{1}{3}} \right)^{-1} \right]^{-1} \cdot (1 + \sqrt[3]{x})$$

2. Find the digits x and y so that the following equality is true.

$$3 \cdot \overline{xxxxx} = \overline{yyyyy} - y$$

3. Consider the rhombus $ABCD$. The circle of radius R circumscribed around the triangle ABD passes through the center of the circle inscribed in the triangle BCD . Express the area of the rhombus in terms of R .
4. For which value of the parameter k will one root of the equation

$$(k^2 - 5k + 3)x^2 + (3k - 1)x + 2 = 0$$

be twice the other root?

5. In the triangle ABC the angle at B is $\beta = 60^\circ$. We draw the angle bisectors from A and C ; they meet at O , and they meet the opposite sides at D and E . Show that $OD = OE$.
6. Solve the following inequality over the real numbers.

$$0.2^{\log_{0.7} \frac{2x+1}{2x-3}} < 1$$

7. In a tetrahedron two opposite edges are x units long, and the other edges are 1 unit long. Express the volume of the tetrahedron as a function of x . For which x is the volume largest? What is the largest volume?
8. Prove that the medians AA_1 and BB_1 of the triangle ABC are perpendicular if and only if, with the usual notation,

$$\cot \gamma = 2(\cot \alpha + \cot \beta)$$