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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

$$\text{a) } \frac{(x-1)(x^3+x^2+x+1)}{(x+1)(x^3-x^2+x-1)} = 1 \quad \text{b) } \sqrt{\tan^2 x} = -\frac{1}{\cot x} \quad \text{c) } \frac{\log_x 7}{\log_x 3} = \frac{\log_3 5}{\log_7 5}$$

2. The diagonals of a trapezoid are perpendicular. Its parallel sides are 13 and 39 units long, and one of its legs is $\sqrt{369}$ units long. Find the other leg, the area, and the height of the trapezoid.

3. Find the values of the parameter m for which the equation

$$(m+1)x^2 + (2m+1)x - 2 = 0$$

has two different real roots, both less than -1 .

4. The first term of a sequence is 1, the sixth term is 51, and the sum of the first three terms is 23; the differences of neighboring terms are consecutive terms of an arithmetic sequence. Find the first three terms of the sequence.

5. Two sides of a triangle, a and c , are given, with $\frac{c}{2} < a < c$, and

$$\frac{\sin \alpha}{\sin \beta} = \frac{\cos(\alpha + \gamma)}{\cos(\beta + \gamma)}$$

where α is the angle opposite a , γ is the angle opposite c , and β is the angle opposite the third side b . Express the side b in terms of a and c .

6. Two circles are externally tangent to each other. We rotate the segment of one of their common external tangents between the points of tangency around the line through the centers of the circles. The lateral surface area of the resulting frustum of a cone is 576π square units, and the radius of one of the circles is 16 units. What is the radius of the other circle?

7. The circle k is tangent to the y -axis, and is tangent to the line $3x - 4y = 48$ at the point E of the line with x -coordinate $x_1 = 8$. Find the equation of the circle.

8. Find the range of the expression

$$\frac{4}{4 \cdot 2^{2x} - 8 \cdot 2^{x-1} + \frac{11}{3}}$$

for $-2 \leq x \leq 1$.