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In memory of Károly Kőváry, principal, mathematics teacher, my former colleague

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance. The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

$$\begin{array}{ll} \text{a) } \sqrt{5x-6} = 2 + \sqrt{x-2} & \text{b) } \sqrt{5x-5} = 2 + \sqrt{x-2} \\ \text{c) } \sqrt{5x-4} = 2 + \sqrt{x-2} & \text{d) } \sqrt{5x-14} = 2 + \sqrt{x-2} \end{array}$$

2. The diagonals of the convex quadrilateral $ABCD$ are perpendicular, and the diagonal BD bisects the diagonal AC . Also $\angle ABC = 120^\circ$, $AC = 4\sqrt{3}$, and $BD = 8$. Find the area, the sides, and the angles of the quadrilateral.

3. Prove that if $-1 < a < 0$ or $0 < a < 1$, then $\frac{1}{a^2} + \frac{1}{1-a^2} \geq 4$.

4. From which point of the segment joining the centers of the circles $x^2 + y^2 = 9$ and $(x-4)^2 + (y-8)^2 = 1$ can we draw common tangents to the two circles? Find the equations of these tangent lines.

5. Find the value of $\frac{y}{x}$, if

$$\text{a) } \lg^2 y + \lg^2 x - 2 \lg y \cdot \lg x - \lg y + \lg x = 2; \quad \text{b) } \cos \frac{y+2x}{2x} = \cos \frac{y-2x}{2x}.$$

($\lg$ denotes the base 10 logarithm.)

6. Find the set of ordered pairs (x, y) that satisfy the following system of equations.

$$\sqrt{x^2 - 4x + 4} + \sqrt{y^2 - 6y + 9} = 1, \quad |x-2| + y = 4$$

7. Through the vertices of an equilateral triangle we draw three parallel lines. The distances of the middle line from the other two are 1 and 4. Find the side of the equilateral triangle.

8. a) Prove the inequality $a^2 + b^2 + c^2 \geq ab + bc + ca$, where a, b, c are real numbers.

b) Prove that the function $f(x) = (x+a)(x+b) + (x+b)(x+c) + (x+c)(x+a)$ defined on the real numbers has a zero for all real values of a, b , and c .

c) Prove that the function $g(x) = b^2x^2 + (b^2 + c^2 - a^2)x + c^2$ defined on the real numbers has no zero if a, b , and c are the sides of a triangle.