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In memory of György Ágoston (Miskolc, Szeged), Károly Kellner, Tamás Holczer, and my other friends who were killed in the Holocaust

translated from: KoMaL, April 2004, page 214

Pdf file: KoMaL, April 2004, page 214

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance. The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

a) $\sin\left(3x - \frac{\pi}{3}\right) = \sin\left(2x + \frac{\pi}{6}\right)$

b) $\sin\left(3x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} - 2x\right)$

c) $\cos\left(3x - \frac{5\pi}{6}\right) = \sin\left(2x + \frac{\pi}{6}\right)$

d) $\cos\left(\frac{5\pi}{6} - 3x\right) = \cos\left(2x - \frac{\pi}{3}\right)$

2. Solve the equation $\sqrt{x} + \sqrt{x - 4a + 4} = 2$ over the real numbers, where a is a real parameter.

3. The altitude to the hypotenuse AB of the right triangle ABC is CD . The radii of the circles inscribed in the triangles ACD and BCD are $\varrho_1 = 3$ and $\varrho_2 = 4$. Find the radius of the circle inscribed in the triangle ABC .

4. The equation of the median through the vertex B of the triangle ABC is $3x + 5y = 10$, and the equation of the median through C is $3x - 2y = 3$. One vertex of the triangle is $(-3, 1)$. Find the coordinates of the other two vertices.

5. Solve the following equations over the real numbers.

a) $\lg^2(5 - x) + 2\lg^2(x + 1) = 3\lg(5 - x) \cdot \lg(x + 1)$

b) $3^{2x+1} + 3^{2-2\sqrt{x}} = 4 \cdot 3^{1+x-\sqrt{x}}$

c) $2\sin^4 x - 2\sin^2 x \cos^2 x + 4\cos^4 x = 1$

6. For which integers n is the value of the expression

$$\frac{18nx + 36x + 12n + 6n^2}{3n^2x + 15nx + 18x + 6n + 5n^2 + n^3}$$

an integer, if x can be any integer?

7. For which values of the real parameter p is the sum of the squares of the real roots of the equation $x^2 + (2p - 1)x + \frac{3}{2}p^2 - 5p + \frac{1}{4} = 0$ smallest, and for which is it largest? What are the smallest and largest values?

8. The points K on the side AB and L on the side BC of the triangle ABC satisfy $3AK = KB$ and $BL = LC$. The segments AL and CK meet at Q .

a) What fraction of the area of the triangle ABC is the area of the triangle AQB , and of the triangle BQC ?

b) In what ratio does Q divide the segments AL and CK ?