

Problem Set II - 2004 (April 2004 supplement)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. For which values of x is it true that

a) $9 - (x - 1)^2 > 0$; b) $2|\sin x| = \frac{3}{\sqrt{3} - 1} - \frac{\sqrt{3}(\sqrt{3} + 1)}{2}$; c) both a) and b) hold?

2. The diagonal of a regular pentagon is 8 cm long.

- a) What is its side? b) What is its area? c) Into what parts is one of its diagonals divided by the two diagonals that cross it?

3. A car travels at a constant speed on a straight road. At two o'clock and at three o'clock the driver sees kilometer signs with two-digit numbers whose digits are the same, in the two different orders. At four o'clock the car passes a sign with a three-digit number. This looks like the number seen two hours earlier, but with a zero between its two digits. Find the speed of the car.

At four o'clock the car turns around and goes back to the sign seen at two o'clock. At what speed must it go so that its average speed for the whole round trip is 20% more than between two and four o'clock?

4. A bank offers two kinds of investments.

The first: the part of the deposit below 1 million Ft earns 5.5% interest, the part between 1 million Ft and 3 million Ft earns 6%, and the part above 3 million Ft earns 7%, credited yearly.

The second: 1.6% interest every three months, credited quarterly.

Compare the two investments as a function of the deposit.

5. Prove the following statements.

- a) Every prime number greater than 3 has a neighbor divisible by 6.
b) $n^4 + 4$ is a composite number if n is an integer greater than 1.

6. Six boys and four girls study in an English language group. The teacher calls on three students. What is the probability that both boys and girls are among the students called on?

7. What is the range of the function $f(x) = \frac{1}{\sin^4 x + \cos^4 x}$ defined on the real numbers? Find the places of the maximum and the minimum.

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8. Every member of a group speaks at least one of French and German. 70% of them speak German and 60% speak French.
- a) What is the probability that a randomly chosen person cannot answer in German?
 - b) How many people speak only one language, if 12 people do not speak French?
 - c) What is the answer to a), under the condition of b), if the group grows by 6 members who speak both languages?
9. In a department store, two rectangular sales areas of equal area (“A” and “B”) are to be created. The available space and operating conditions require that
- the width of area “A” is 8 m less than its length;
 - the width of area “B” is 1 m less than the width of area “A”;
 - the length of area “B” is as small as possible.
- Give the dimensions of the two areas.