Advanced Level Practice Set - 2004 (December 2004)

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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.
The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Show that there is no pair of real numbers (x, y) for which

$$\left(x^2+5\right)^2 = 25 - |y-3|, \qquad 21x + 63y = 188$$

(11 points)

2. Consider the function $f(x) = ax^2 + (2a+b)x - \left(b^2 - b - a\right)$ defined on the real numbers.
   a) Show that if a and b are integers, then $f(-2) + f(-1) + f(0) + f(1) + f(2)$ is divisible by 5.
   b) Prove that if a is positive, then $f(x)$ has a zero.

(12 points)

3. How many positive terms less than 2 does the sequence $a_n = -5 + \log_2(n+4)$ have?

(14 points)

4. On March 25, 2005, Ágnes deposits 600 000 Ft in a bank where the yearly interest is 8%, and the interest is computed at the end of the calendar year. How much will she have on March 25, 2006?

(14 points)

Part II

5. Three lines are given by their equations:

$$e:\ x - \sqrt{3}y = 0, \qquad f:\ \sqrt{3}x - y = 0, \qquad g:\ \sqrt{3}x + y - 10 = 0$$

   a) Show that the three lines determine a right triangle.
   b) What is the radius of the circle circumscribed around the triangle?
   c) Find the coordinates of the center of the inscribed circle.

(16 points)

6. Consider the function $f(x) = 2x^6 - 3x^4 + x^2$ defined on the real numbers.
   a) Find the exact value of $f\left(\tan\frac{\pi}{3}\right)$.
   b) Show that $f(\sin\alpha) + f(\cos\alpha)$ does not depend on α.

(16 points)

7. In a high school, 75% of the students do their math homework. A journalist wants to talk with five randomly chosen students of this school about their study habits.

   a) What is the probability that he chooses only students who have done their homework?

   b) What is the probability that at least three of the five chosen students have done their homework?

   c) In class 12.c of the school, with 20 students (where the situation is a little better than the school average), 16 wrote their homework. The class has 3 girls, who always do their homework. If we choose 4 boys and 1 girl from this class, what is the probability that exactly two of those chosen have not done their homework?

   (16 points)

8. Walking from the bank of a 27 m wide river perpendicular to the bank, we measured the depth of the water every 3 meters. We got the following data, in centimeters: 60, 120, 150, 240, 210, 180, 90, 30.

   a) What is the area of the cross section, in m^2, if we assume that the river bed is a straight line between two measuring points?

   b) Find the mean and the median of the measured data.

   c) What is the average depth of the river along the line of the measurement?

   d) How much water flows through the cross section of the river in 1 hour, if the speed of the river is 85 m/min?

   (16 points)

9. Among the convex quadrilaterals with the same perimeter, for which is the product of the sums of the opposite sides largest? Find this largest value.

   (16 points)

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