Problem Set - 2004 (February 2004)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Consider the right circular cylinders in which the sum of the radius of the base and the height is 10 cm. Which of these cylinders has the largest lateral surface area? What is the volume of the cylinder in this case?

2. Solve the following system of linear equations, where a is a real parameter.

$$(a-1)\,x+3y=3a, \qquad x+(a+1)\,y=2$$

3. The area of an isosceles trapezoid is $144\sqrt{3}$ cm^2, and its legs are equal to the radius of the circle circumscribed around the trapezoid. Find the height, the diagonal, and the midline of the trapezoid. Can the parallel sides of the trapezoid be determined?

4. Solve the following equations over the real numbers.
   a) $\sqrt{4\cdot 2^{2x}-12\cdot 2^x+9}=3-2^{x+1}$ b) $\sqrt{2\cos 2x-4\cos x+3}=1-2\cos x$

5. S_n, S_k, and S_{n+k} are the sums of the first n, k, and $n+k$ terms of an arithmetic sequence. Prove that

$$(n+k)\,(S_n-S_k)=(n-k)\cdot S_{n+k}$$

6. Consider the function $x\mapsto f\,(x)$ defined on the interval $[-4,4]$, where

$$f\,(x)=\frac{|x-3|+|x+1|}{|x+3|+|x-1|}$$

   Find the maximum and the minimum of the function, and the values of x where they are taken.

7. Prove that if α and β are two angles of a triangle and

$$\sin\,(\alpha-\beta)=\sin^2\alpha-\sin^2\beta$$

   then the triangle is a right triangle or an isosceles triangle.

8. Notice that

$$44-8=6^2, \qquad 4444-88=66^2, \qquad 444\,444-888=666^2$$

   Prove that

$$\underbrace{44\ldots4}_{2n\text{ digits}}-\underbrace{88\ldots8}_{n\text{ digits}}=\underbrace{66\ldots6}_{n\text{ digits}}{}^2$$

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