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In memory of my parents and my brother

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Pdf file: KoMaL, January 2004, page 17

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

a) $(2y - 1)(y - 1) = 0$

b) $2^{2x+1} - 3 \cdot 2^x + 1 = 0$

c) $2 \sin^2 x - 3 \sin x + 1 = 0$

d) $2 \cdot \log_2^2 x - \log_2 x^3 + 1 = 0$

2. Solve the following inequalities over the real numbers.

a) $2y^2 - 3y + 1 > 0$

b) $2^{2x+1} - 3 \cdot 2^x + 1 > 0$

c) $2 \sin^2 x - 3 \sin x + 1 < 0$

d) $2 \cdot \log_2^2 x - \log_2 x^3 + 1 > 0$

3. The endpoints of one leg of an isosceles right triangle are $A(-2, 3)$ and $B(1, 2)$. Find the coordinates of the third vertex of the triangle.

4. Find p and q , if one root of the equation $x^2 + px + q = 0$ is 2, and the other root is three times the discriminant of the equation.

5. In the triangle ABC , $AC = 10$, $BC = 15$, and $\angle ACB = 60^\circ$.

a) What is the radius of the semicircle whose diameter lies on the side AB and which is tangent to the other two sides?

b) What is the radius of the circle that is tangent to this semicircle and to the sides AC and BC ?

6. The difference of an arithmetic sequence is $\frac{2}{3}$, the sum of the first n terms is $\frac{8}{3}$, and the sum of the first $(n + 3)$ terms is $\frac{44}{3}$. Find n and the first term of the sequence.

7. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$,

$$f(x) = \frac{(x+5)^2 - (x-5)^2}{(2x+5)^2 + (2x-5)^2}$$

Find the largest and smallest values of the function, and the values of x where they are taken.

8. The diagonals AC and BD of the convex quadrilateral $ABCD$ meet at K . The areas of the triangles ABK , BCK , CDK , and DAK are t_1 , t_2 , t_3 , and t_4 . Prove that the sides AB and DC of the quadrilateral are parallel if and only if the area T of the quadrilateral is

$$T = (\sqrt{t_1} + \sqrt{t_3})^2$$