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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations.

$$\text{a) } \frac{x^3 - 4x}{2 - x} = 0 \qquad \text{b) } \sqrt{\frac{x^3 - 4x}{2 - x}} = \frac{\sqrt{3}}{2} \qquad \text{c) } \lg \frac{x^3 - 4x}{2 - x} = 0$$

2. Prove that if $a + 1$ is divisible by 6, then $n(an^2 + 1)$ is divisible by 6 for every integer n .
3. In a circle of radius 5 cm, the chord AB is equal to the side of the regular octagon inscribed in the circle, and the chord BC is equal to the side of the regular hexagon inscribed in the circle. How long is the segment AC ?
4. For which values of the parameter a does the equation

$$a^2(x - 1) = 4(ax - x - 1)$$

have an integer root?

5. Two vertices of the triangle ABC of area 13 are $A(3, -1)$ and $B(-1, 5)$. Find the coordinates of the vertex C so that the perimeter of the triangle is as small as possible.
6. On two neighboring sides of a rectangle with perimeter 60 cm we draw semicircles outward, with the sides as diameters. How should we choose the sides of the rectangle so that the resulting figure
- a) has the smallest perimeter; b) has the largest area?
7. The angles α and β of a triangle satisfy

$$(1 + \cot \alpha)(1 + \cot \beta) = 2 \cot \alpha \cot \beta$$

Find the third angle of the triangle.

8. We join the vertex A of the convex pentagon $ABCDE$ with the midpoints of the diagonal BE and of the sides BC , CD , and DE . On these four segments, the points of trisection farther from A determine a quadrilateral whose area is one fifth of the area of the pentagon. What fraction of the area of the pentagon is the area of the triangle ABE ?