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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Solve the equation

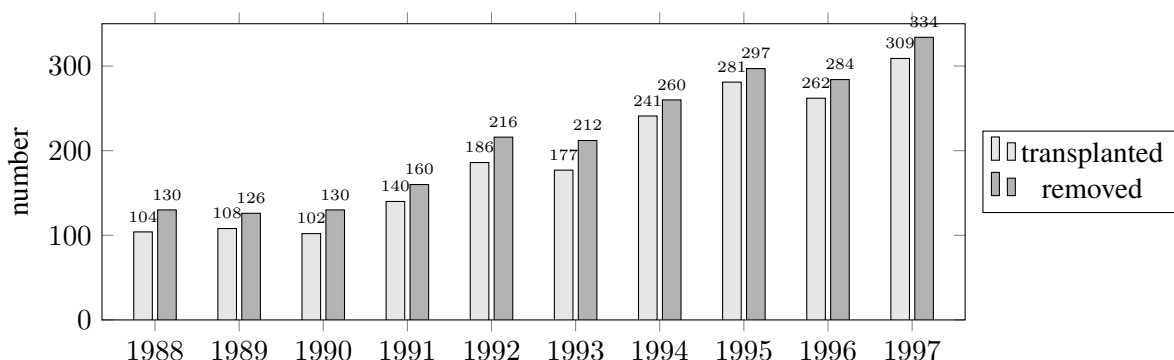
$$x \cdot \left(\frac{x-3}{3} \right)^2 + \frac{1}{3} (x-2)(x+2) = \frac{1+(x-1)^3}{9}$$

2. We roll a die 6 times and record the results.

- How many different sequences can we get?
- How many different sequences can we get, if there is a 1 in the first place and nowhere else?
- What is the probability that the number in the first place is different from all the others?

3. A math teacher says that she composed the test well if the average of the grades is between e and π ($e = 2.718\dots$ and $\pi = 3.141\dots$). In a class of 35, she has not yet looked at the work of the best student, and at that point the average was 2.68. How could the best student's test turn out, if the teacher could say about herself that she composed the test well? (Grades are integers from 1 to 5.)

4. Consider the following diagram. It shows the numbers of transplanted and removed kidneys over a 10-year period.



- In which year was the largest percentage of the removed kidneys transplanted?
- Show in a diagram the ratio of transplanted to removed kidneys in the first three and the last three years.
- Can any conclusion be drawn from the resulting diagram?

Part II

5. One diagonal of a trapezoid makes a 30° angle with the base. The two diagonals are perpendicular.

- How long are the two diagonals, if the parallel sides are 6 and 4 units long?
- What is the area of the trapezoid?
- Find the perimeter of the trapezoid.

6. Consider the functions $f(x) = 13 - (x - 1)^2$ and $g(x) = 4|x + 1|$ defined on the real numbers.
- a) Solve the inequality $f(x) \geq g(x)$ graphically.
 - b) What is the area of the region bounded by the graphs of f and g ?

7. Prove that the difference of the roots of the equation

$$5x^2 - 2(5k + 3)x + 5k^2 + 6k + 1 = 0$$

is the same for every value of k .

8. In the triangle ABC , $\beta - \gamma = 90^\circ$. Prove that the line of the altitude from the vertex A is also tangent to the circle circumscribed around the triangle.
9. Among the isosceles triangles with perimeter 50 cm, choose the one for which the sum of the areas of the squares drawn on its sides is as small as possible.