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From 2005, the Hungarian school-leaving exam (érettségi) in mathematics can be taken at two levels: standard level and advanced level. The advanced-level exam also serves as the university entrance exam in mathematics.

The written part of the advanced-level exam takes 240 minutes. Part I consists of 4 problems worth 51 points in all. Part II consists of 5 problems worth 16 points each, of which students choose and solve 4, for 64 points. So there is a total of 115 points possible.

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 240 minutes to solve and present the problems.

Part I

1. Find the coordinates of the vertices of the square whose diagonals are parallel to the coordinate axes and three of whose vertices are on the parabola $x^2 - 6x - 5y + 4 = 0$.
 - a) What is the area of the circle inscribed in the square?
 - b) How many lattice points does the closed disk have?

(11 points)

2. Solve the following equation over the real numbers.

$$1 - x^2 - x^4 = 4^{\sin^2 x}$$

(12 points)

3. The following table shows the change of the consumer price index in some countries between 1995 and 1999:

Country	change compared to the previous year, %				
	1995	1996	1997	1998	1999
Austria	2.3	1.5	1.3	1	0.6
France	1.7	2	1.2	0.8	0.6
Poland	27.8	19.9	14.8	11.6	7.3
Hungary	28.2	23.6	18.3	14.3	10
Great Britain	3.4	2.4	3.2	3.4	1.6
Germany	1.8	1.4	1.9	1	0.6

- a) Explain what the numbers 1 and 0.6 in the last two columns for Austria mean.
 - b) If someone hid 34 000 forints in their mattress in 1994, how many forints did its purchasing power correspond to in 1999?
 - c) In 1999, when her grandchild graduates, Grandma wants to give 200 000 forints as a graduation present. How much money did she put in 1994, upon hearing the news of the successful entrance exam, into a bank that, as it turned out later, paid interest exactly equal to the consumer price index between 1994 and 1999?
4. The base of the pyramid with apex D is the equilateral triangle ABC . The base edges are 18 cm, and the lateral edges are $6\sqrt{6}$ cm long. We pass a plane parallel to the edge AD through the midpoints of the edges AB and AC .
 - a) Find the area of the cross section.
 - b) What angle does the cutting plane make with the plane of the base?

(14 points)

(14 points)

Part II

5. In a group of 16 students, the chemistry average was 3.81 (rounded to two decimal places). We know that nobody failed (grades are from 1 to 5, and 1 is failing).
- a) At most how many students could have received a 2?
 - b) Is it certain that someone received a 5?
 - c) In how many ways could there have been exactly 11 grades of 4?
 - d) Is it true that if the mode is 4, then the median is also 4?

(16 points)

6. The radius of the inscribed circle, the radius of the circumscribed circle, and the perimeter of a right triangle are three consecutive terms of a geometric sequence. Prove that then the sum of the tangents of the two acute angles is 4.

(16 points)

7. The distance of a water tower (P) from the village A has to be determined with the help of a map of scale 1 : 250 000. For this, the distances of two villages (B and C) from A are known from the map: $AB = 17.2$ cm and $AC = 20.0$ cm. We also measured that

$$\angle APB = 83^\circ, \quad \angle APC = 130^\circ, \quad \angle BAC = 65^\circ$$

What is the distance AP in reality?

(16 points)

8. Find the cubic function with the following properties:

- a) it touches the curve $x^2 + y = 4$ from inside at $x = 0$;
- b) the two curves meet at $x = 2$;
- c) the area enclosed by the two curves on the interval $[0, 2]$ is $\frac{4}{3}$.

Describe the behavior of the cubic function.

(16 points)

9. A group of 9 people gets on a suburban train of three cars, but in the big crowd every member of the group only cares about getting on one of the cars, not about which cars the others got on.

- a) What is the probability that 3 members of the group got on each of the three cars?
- b) What is the probability that nobody from the group got on at least one of the three cars?
- c) What is the probability that at most one person from the group got on at least one of the three cars?

(16 points)